UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2023

MATHEMATICS — MDC

Paper: CC-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise:
https://palmaths.com/cu-pyqs/2023/maths-mdc-sem-1-cc-1/

Write the answers to each Group in a separate answer-book.

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual meanings.

Group - A

[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) If $\lim_{x\to 0} \dfrac{\sin 2x + p \sin x}{x^3}$ be finite, find the value of p.

(b) Evaluate $\displaystyle\int_0^{\pi/4} \tan^5 x \, dx$.

(c) If $y = \dfrac{x}{x+1}$, find $y_5(0)$ where y_n is the nth derivative of y w.r.t. x.

(d) If $I_n = \displaystyle\int_0^{\pi/2} x^n \sin x \, dx$ $(n > 1)$, prove that $I_n + n(n-1)I_{n-2} = n\left(\dfrac{\pi}{2}\right)^{n-1}$.

(e) If $y = x^{2n}$ (n : a positive integer), then show that $y_n = 2^n[1 \cdot 3 \cdot 5 \cdots (2n-1)]x^n$.

(f) Evaluate $\lim_{x\to\infty} \left(\sqrt{x^2 + 2x} - x\right)$.

(g) Find the interval on which the function $f(x) = x^2 e^{-x}$ is monotonically decreasing.

Answer *any three* questions :

2. (a) Evaluate $\lim_{x\to 0} \dfrac{(1+x)^{1/x} - e}{x}$. **4**

(b) Obtain a reduction formula for $\displaystyle\int \cos^m x \sin nx \, dx$; m, n being positive integers and deduce that

$$I_{m,m} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \cdots + \frac{2^m}{m}\right].$$

2+2

(c) Show that the area bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and the x-axis is $3\pi a^2$, which is equal to three times the area of the generating circle. **4**

(d) Find the length of the perimeter of the curve $r = 2(1 - \cos\theta)$. **4**

(e) Find the values of a and b if $\lim_{x\to 0} \dfrac{x(1 + a\cos x) - b\sin x}{x^3} = 1$. **4**

(f) Find the volume of the solid generated by the revolution of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ about x-axis. **4**

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2}\times 2$

(a) If the radius of a right circular cylinder is 5, axis passes through the point $(1,2,3)$ and is parallel to the straight line $\frac{x-4}{2}=\frac{y-3}{-1}=\frac{z-2}{2}$, then find the equation of the cylinder.

(b) Find the radius of the circle given by $x^2+y^2+z^2-2y-4z-11=0$ and $x+2y+2z=15$.

(c) Find the angle through which the axes must be turned so that the equation $lx+my+n=0$ $(l\neq 0)$ may reduce to the form $ax+b=0$.

(d) Determine the nature and the length of the latus rectum of the conic whose polar equation is $\frac{2}{r}=3-3\cos\theta$.

Answer *any five* questions : 6×5

4. (a) Find the equation of the locus of the point of intersection of two tangents to the parabola $y^2=4ax$ such that the chord of contact subtends a right angle at the vertex.

(b) In an ellipse the normal at an extremity of the latus rectum passes through the extremity of the minor axis. Prove that $e^4+e^2=1$, where e is the eccentricity of the ellipse.

(c) Find the equation of the cylinder whose guiding curve is the ellipse $4x^2+y^2=1$, $z=0$ and generators are parallel to the straight line $\frac{x}{2}=\frac{y}{-1}=\frac{z}{3}$.

(d) Show that the equation of the tangent to the conic $\frac{l}{r}=1+e\cos\theta$ parallel to the tangent at $\theta=\alpha$ is given by $l(e^2+2e\cos\alpha+1)=r(e^2-1)[\cos(\theta-\alpha)+e\cos\theta]$.

(e) Find the nature of the surface given by the equation $3x^2-2y^2-6x-8y-4z=0$.

(f) Show that the perpendiculars from the origin on the generators of the paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2z}{c}$ lie on the surface $\left(\frac{x}{a}\pm\frac{y}{b}\right)(ax\pm by)+2z^2=0$.

(g) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as the great circle.

(h) Reduce the equation $x^2-6xy+y^2-4x-4y+12=0$ to its canonical form and hence determine the nature of the conic.

(i) Find the equations to the generating lines of the paraboloid $(x+y+z)(2x+y-z)=6z$ which passes through the point $(1,1,1)$. Hence, find the angle between these generators.

Group - C
[Vector Analysis]
(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Find, by vector method, the volume of the tetrahedron $ABCD$ with vertices $A(1,1,-1)$, $B(3,-2,-2)$, $C(5,5,3)$ and $D(4,3,2)$.

(b) Find the projection of the vector $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

(c) A particle, acted on by constant force $4\hat{i} + 2\hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j}$ to $2\hat{i} - \hat{j} + 3\hat{k}$. Find the work done by the force.

(d) Find the value of the constant d such that the vectors $(2\hat{i} - \hat{j} - \hat{k})$, $(\hat{i} + 2\hat{j} - 3\hat{k})$ and $(3\hat{i} + d\hat{j} + 5\hat{k})$ are coplanar.

(e) A force $5\hat{i} + 2\hat{j} - 3\hat{k}$ is applied at the point $(1, -2, 2)$. Find the value of the moment of the force about the origin.

(f) Show that the perpendicular distance of the point $(2\hat{i} + 3\hat{j} - \hat{k})$ from the plane $\vec{r} \cdot (4\hat{i} - 3\hat{j} + \hat{k}) = 18$ is $\dfrac{20}{\sqrt{26}}$ units.

(g) If $\vec{u} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{v} = (2t - 3)\hat{i} + \hat{j} - t\hat{k}$, then show that $\dfrac{d}{dt}(\vec{u} \cdot \vec{v}) = -6$ at $t = 1$.

Answer *any three* questions :

6. (a) By vector method, put the equation of the plane $5x - 6y + 7z = 8$ in normal form and then find the equation of the plane passing through $(2, 3, 4)$ and parallel to the plane $5x - 6y + 7z = 8$. **2+2**

(b) Prove that a necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a} \cdot \dfrac{d\vec{a}}{dt} = 0$. **4**

(c) If $\vec{r} = 5\cos t\,\hat{i} + 5\sin t\,\hat{j} + 7t\,\hat{k}$, then find the value of $\left[\dfrac{d\vec{r}}{dt}\ \dfrac{d^2\vec{r}}{dt^2}\ \dfrac{d^3\vec{r}}{dt^3}\right]$. **4**

(d) A rigid body is spinning with an angular velocity 3 radians per second about an axis parallel to $2\hat{i} - \hat{j} + \hat{k}$ and passing through the point $\hat{i} + 2\hat{j} - 3\hat{k}$. Find the velocity of the particle at the point $-4\hat{i} + \hat{j} + \hat{k}$. **4**

(e) Show that if the straight lines $\vec{r} = \vec{a} + u\vec{\alpha}$ and $\vec{r} = \vec{b} + v\vec{\beta}$ intersect, then $(\vec{a} - \vec{b}) \cdot \vec{\alpha} \times \vec{\beta} = 0$ but $\vec{\alpha} \times \vec{\beta} \neq \vec{0}$. **4**

(f) Prove that for any vector $\vec{a}$, $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$. **4**

PALMATHS