UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2023

MATHEMATICS — MINOR

Paper: MN-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual meaning.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Evaluate : $\lim_{x\to 0} \dfrac{x^2 \sin\left(\frac{1}{x}\right)}{\sin x}$. Is L'Hospital's rule applicable here? Explain.

(b) Find $\dfrac{dy}{dx}$: $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$, when $\theta = \dfrac{\pi}{2}$.

(c) If $y = (x^2 - 1)^n$, then show that $(x^2 - 1)y_2 - 2(n-1)xy_1 - 2ny = 0$.

(d) Evaluate $\int x^3 e^x dx$ using the reduction formula $\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx$.

(e) Find the curve whose parametric equation is $x = 2\sin^2 t$, $y = 3\cos^2 t$, $t \in \left[0, \dfrac{\pi}{2}\right]$, where t is a parameter.

(f) Show that the length of the circumference of the circle $x^2 + y^2 = 36$ is 12π.

(g) Find the area bounded by the curve $y = x^2$ and the line $y = 2x$.

Answer *any three* questions : 4×3

2. (a) Find the values of a and b in order that $\lim_{x\to 0} \dfrac{x(1 + a\cos x) - b\sin x}{x^3} = 1$.

(b) Find the nth order derivative of the function $y = \sin(ax + b)$.

(c) If $y = \tan^{-1} x$, then show that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

(d) Find the Reduction formula for $\int \tan^n x\, dx$, hence evaluate $\int \tan^4 x\, dx$.

(e) Find the area enclosed by the curve $x = a\cos^3\theta$, $y = b\sin^3\theta$.

(f) Find the volume of the solid generated by revolution of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ about the X-axis.

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

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(a) Transform the equation $x^2 - y^2 = 25$ when the axes are rotated through $45°$.

(b) Find the point on the parabola $\frac{l}{r} = 1 - \cos\theta$ which has the smallest radius vector.

(c) Determine whether the point $(2, 3, 4)$ is an outside or an inside point with respect to the sphere : $x^2 + y^2 + z^2 - 2x - 4y - 6z - 25 = 0$

(d) Find the equation of the cone whose vertex is the point $(0, 0, 0)$ and whose base is the curve $x^2 + y^2 = 4, z = 2$.

Answer *any five* questions : 6×5

4. (a) Reduce the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ to its canonical form and hence determine the nature of the conic.

(b) If the straight line $r\cos(\theta - \alpha) = p$ touches the parabola $\frac{l}{r} = 1 + \cos\theta$, then show that $p = \frac{1}{2}l\sec\alpha$.

(c) Show that the locus of the point of intersection of a pair of perpendicular tangents to an ellipse is a circle.

(d) A plane passing through a fixed point (α, β, γ) cuts the coordinate axes at A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 2$.

(e) Find the equation of sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 5 = 0, x - 2y + 3z + 1 = 0$ is a great circle.

(f) Find the equation of the right circular cone whose vertex is the origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi-vertical angle of $30°$.

(g) Find the equation of the right circular cylinder of radius 2, whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$.

(h) Find the equations to the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$, which pass through the point $(2, 3, -4)$.

(i) Show that the quadric surface given by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$ is an ellipsoid whose centre is $(1, -2, 1)$.

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Determine the value of the constant d such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + d\hat{k}$ and $2\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar.

(b) Find the equation of the line passing through the point $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $3\hat{i} + 4\hat{j} + 5\hat{k}$.

(c) Prove that, for any proper vector $\vec{a}$, $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

(d) If $\vec{A} = 5t^2\hat{i} + t\hat{j} + t^3\hat{k}$ and $\vec{B} = \sin t\,\hat{i} - \cos t\,\hat{j}$, find $\frac{d}{dt}(\vec{A} \cdot \vec{B})$.

(e) Show that $\left[\hat{i}+\hat{j}\ \ \hat{j}+\hat{k}\ \ \hat{k}+\hat{i}\right] = 2$.

(f) Let $\vec{a}$ and $\vec{b}$ be two given vectors and let k be a given scalar. Find all vectors $\vec{r}$ such that $\vec{a}\times\vec{r} = \vec{b}$ and $\vec{a}\cdot\vec{r} = k$.

(g) O, Q, R and S are four points, no three of which are collinear. If $\vec{\beta}, \vec{\gamma}, \vec{\delta}$ be the position vectors of Q, R, S respectively with O as origin, then find the shortest distance between the lines OQ and RS.

Answer *any three* questions : 4×3

6. (a) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the straight line PQ, where Q is the point $(5,0,4)$.

(b) For the curve $\vec{\gamma} = (2a\cos t, 2a\sin t, bt^2)$, deduce that $\left[\dot{\vec{\gamma}}\ \ddot{\vec{\gamma}}\ \dddot{\vec{\gamma}}\right] = 8a^2bt$.

(c) If a vector function $\vec{a}(t)$ possesses constant direction, then show that $\vec{a}\times\dfrac{d\vec{a}}{dt} = \vec{0}$.

(d) A particle, acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$, is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. Find the work done by the force on the particle.

(e) Show by vector method that the angle on a semi-circle is a right angle.

(f) A force $\vec{F} = 3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point $(1,-1,2)$. Find the moment of $\vec{F}$ about the point $(2,-1,3)$. Also find its magnitude.

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