UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — HONOURS

Paper: DSCC-2

(Basic Algebra)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the equation whose roots are squares of the roots of the equation $x^4 - x^3 + 2x^2 - x + 1 = 0$.

(b) If $S_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$, then prove that $S_n > \frac{2n}{n+1}$, if $n > 1$.

(c) Find the general value of i^i.

Answer *any four* questions :

2. (a) Show that the product of all the values of $\left(\sqrt{3} + i\right)^{\frac{3}{5}}$ is $8i$. **5**

(b) (i) Show that the ratio of principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(\log 2) + i\cos(\log 2)$.

(ii) Find the principal value of $\arg z$, where $z = 1 + i\tan\frac{3\pi}{5}$. **3+2**

(c) Using Cauchy-Schwarz inequality, show that

$$\frac{x_1^2}{y_1} + \frac{x_2^2}{y_2} + \cdots + \frac{x_n^2}{y_n} \geq \frac{(x_1 + x_2 + \cdots + x_n)^2}{y_1 + y_2 + \cdots + y_n},$$

where $x_1, x_2, \ldots, x_n$ are real numbers and $y_1, y_2, \ldots, y_n$ are positive real numbers. Applying the above result, prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2},$$

where a, b, c are positive real numbers and $abc = 1$. **2+3**

(d) Obtain the equation whose roots are the roots of the equation $4x^3 - 8x^2 - 19x + 26 = 0$ each diminished by 2. Use Descarte's rule of signs to both the equations to find the exact number of positive and negative roots of the equation. **5**

(e) (i) Find the relation among the coefficients of the equation $ax^3 + 3bx^2 + 3cx + d = 0\ (a \neq 0)$, if its roots are in geometric progression (a, b, c, d are real numbers).

(ii) Apply Sturm's theorem to find the number of real roots of $x^3 - 3x + 2 = 0$. **3+2**

(f) By Ferrari's method, solve the equation $2x^4 + 6x^3 - 3x^2 + 2 = 0$. **5**

Group - B

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) If $S = \{1,2,3,5\}$, then find the equivalence relation corresponding to the partition $\{\{1\},\{2,3\},\{5\}\}$ on S.

(b) Show that $(S, \leq)$ is a poset, where S is the set of all positive divisors of 72 and $a \leq b$ means 'a is a divisor of b' for all $a, b \in S$.

(c) If $a \mid c$, $b \mid c$ and g.c.d $(a,b) = 1$, then show that $ab \mid c$ (where $m \mid n$ means m divides n).

Answer *any four* questions :

4. (a) Find the number of relations, which are reflexive as well as symmetric defined on a set containing n elements. **2+3**

(b) (i) Find a bijective mapping from the set $\mathbb{N}$ onto the set $\mathbb{Z}$.

(ii) Let ρ be an equivalence relation on a set S and $a, b \in S$. Then show that $\text{cl}(a) = \text{cl}(b)$ if and only if $a \rho b$. **2+3**

(c) Prove that for given integers a and b with $b > 0$, there exist unique integers q and r such that $a = bq + r$, where $0 \leq r < b$. **5**

(d) (i) Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions given by $f(x) = |x| + x$ for all $x \in \mathbb{R}$ and $g(x) = |x| - x$ for all $x \in \mathbb{R}$. Find $f \circ g$.

(ii) If p_n is the n-th prime, then prove that $\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n}$ is not an integer. **3+2**

(e) If d is the g.c.d. of two non-zero integers a, b, then prove that there exist two integers u, v such that $d = au + bv$. Are u, v unique? Justify your answer. **3+2**

(f) Solve by Chinese remainder theorem : $x \equiv 1 \pmod{17}$, $x \equiv 1 \pmod{7}$, $x \equiv 4 \pmod{5}$. **5**

Group - C

5. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) If $a \neq b \neq c$, find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}$, where a, b, c are real numbers.

(b) Find λ so that the vector $(0,2,\lambda)$ can be expressed as, $(0,2,\lambda) = a(1,2,2) + b(1,0,-1)$. Also, find the value of a and b.

(c) Find the value(s) of k for which the following system of equations has non-trivial solution :

$$x + y + z = kx, \quad x + y + z = ky, \quad x + y + z = kz.$$

Answer *any four* questions :

6. (a) Using pivoting row reduced algorithm, reduce the following matrix to its row-echelon form :

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$

5

(b) Consider the system of equations :

$$2x - 2y + 2z = 1, \qquad -3x + 6y = -1, \qquad x - 7y + 10z = 2$$

Check the solvability of the equations by applying the row reduced echelon form of the coefficient matrix.

5

(c) For what values of K, the planes $x + y + z = 2$, $3x + y - 2z = K$, $2x + 4y + 7z = K + 2$ intersect in a line? Find the equation of the line in that case. **3+2**

(d) Use elementary row operation to find the inverse of the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 6 & 4 & 1 \end{pmatrix}$. **5**

(e) Show that the vectors, $\alpha = (0, 1, 0)$ and $\beta = (0, 1, 1)$ are linearly independent. Find $L(\alpha, \beta)$. What does $L(\alpha, \beta)$ represent geometrically? Show that $L(\alpha, \beta, \gamma) = \mathbb{R}^3$, where $\gamma = (1, 0, 0)$.

1+1+1+2

(f) Check whether $(0, 12, 20)$ can be expressed as a linear combination of $(1, 2, 3)$, $(2, 4, 6)$ and $(-1, 3, 7)$. Using this result check the solvability of the following system of equations :

$$x + 2y - z = 0, \qquad 2x + 4y + 3z = 12, \qquad 3x + 6y + 7z = 20$$

2+3

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