UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — HONOURS

Paper: DSCC-4

(Ordinary Differential Equations - I and Group Theory - I)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer *any nine* questions.

1. Find the differential equation of the circles having centres on the line $y + x = 0$ and passing through the origin. **5**

2. If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$, then find the values of m and n. Further, for these values of m and n solve the equation. **5**

3. Solve the following differential equation : $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. **5**

4. Reduce the equation $x^3p^2 + x^2yp + a^3 = 0$ to Clairaut's form by the substitution $y = u$ and $x = \frac{1}{v}$ and obtain the complete primitive. **5**

5. Find the general solution of the differential equation $y(4x + y)dx - 2(x^2 - y)dy = 0$. **5**

6. Use D-operator to solve : $\frac{d^2y}{dx^2} - y = x\sin x + (1 + x^2)\, e^x$. **5**

7. Solve using the method of undetermined coefficient $(D^2 - 2D + 3)y = x^2 + \sin x \left(D \equiv \frac{d}{dx}\right)$. **5**

8. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + a^2y = \tan ax$. **5**

9. Transform $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - y = 3x^3\cos(\log x)$ into a linear equation with constant coefficient by a suitable substitution and hence solve it. **2+3**

10. Verify that the equation $(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x + 3)\frac{dy}{dx} + 2y = (x + 1)e^x$ is exact and hence solve it. **2+3**

11. Solve the following by changing the independent variables : $(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos\log(1 + x)$. **5**

12. Factorise the differential expression and obtain the complete primitive : $(x + 2)y_2 - (2x + 5)y_1 + 2y = (x + 1)e^x$. **2+3**

13. Solve the system of simultaneous equation : $Dx - 7x + y = 0$, $Dy - 2x - 5y = 0$, where $\left(D \equiv \frac{d}{dt}\right)$. **5**

14. Show that the equation of the curve, whose slope at any point (x, y) is equal to $xy(x^2y^2 - 1)$ and which passes through the point $(0, 1)$ is $x^2y^2 = 1 - y^2$. **5**

Group - B
(Group Theory)

Answer *any six* questions.

15. (a) Prove that the set of all odd integers forms a commutative group with respect to '$*$' defined by $a * b = a + b - 1\ \forall a, b \in D$.

(b) In a group $(G, *)$ each element is its own inverse. Prove that the group is commutative. **3+2**

16. Let H and K be subgroups of a group G. Then prove that HK is a subgroup of G if and only if $HK = KH$. **5**

17. Let $(G, \circ)$ be a group of H be a non-empty finite subset of G. Then prove that $(H, \circ)$ is a subgroup of $(G, \circ)$ if and only if $a \in H, b \in H \Rightarrow a \circ b \in H$. **5**

18. What is centre of a group? Prove that it forms a subgroup of the group. **1+4**

19. Prove that a non-commutative group of order 10 must have a subgroup of order 5. **5**

20. (a) Let $G = \langle a \rangle$ and $O(G) = 30$, then find the order of the cyclic subgroup generated by a^{18}.

(b) Let G be an abelian group of order 6, containing an element of order 3. Prove that G is a cyclic group. **2+3**

21. (a) Show that every group of prime order is cyclic.

(b) Give an example of a non-cyclic group in which all proper subgroups are cyclic subgroups. **3+2**

22. (a) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 8 & 5 & 6 & 4 & 7 & 1 \end{pmatrix}$ on S_8 as a product of transpositions and find the order.

(b) Find the largest order of an element in the group S_6. **(2+1)+2**

23. (a) Let $(G, \circ)$ be a group and $a, b \in G$. If $o(a) = 3$ and $a \circ b \circ a^{-1} = b^2$, find $o(b)$ if $b \neq e$.

(b) Find all the distinct cosets of the subgroup $H = \{e, (1,2,3), (1,3,2)\}$ in the subgroup S_3. **4+1**

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