UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — HONOURS

Paper: SEC-3

(Linear Programming and Rectangular Games)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any five* questions : 3×5

(a) Solve graphically the LPP :

$$\text{Maximize } Z = 5x_1 - 2x_2$$
$$\text{subject to } 5x_1 + 6x_2 \geq 30,\ \ 9x_1 - 2x_2 = 72,\ \ x_2 \leq 9,\ \ x_1, x_2 \geq 0$$

Hence, identify the redundant equation, if there be any.

(b) Define convex set. Show that the set $X = \{(x, y) \in E^2 : xy \leq 1, x \geq 0, y \geq 0\}$ is not a convex set.

(c) What are the extreme points of the convex set $X = \{(x, y) \mid |x| \leq 1, |y| \leq 1\}$? Give an example of a convex set which has no extreme point.

(d) For the system of equations

$$2x_1 - x_2 + 3x_3 = 3, \qquad -6x_1 + 3x_2 + 7x_3 = -9$$

show that $(2, 1, 0)$ is not basic though it is feasible solution. Reduce the feasible solution to a basic feasible solution.

(e) Find a basic feasible solution of the system of equations : $x_1 + 2x_2 + x_3 = 4, 2x_1 + x_2 + 5x_3 = 5$.

(f) Find the dual of the following linear programming problem :

$$\text{Maximize } Z = 3x_1 + 2x_2 \text{ subject to } 2x_1 + x_2 \leq 5,\ x_1 + x_2 \leq 3,\ x_1, x_2 \geq 0$$

(g) Find an initial basic feasible solution, by North-West method, of the following transportation problem :

	1	2	3	4	
X	4	2	5	3	6
Y	5	4	3	2	13
Z	1	4	6	5	9
	7	8	5	8	

(h) For what value of p, the game with the following pay-off matrix is strictly determinable?

$$\text{Player A} \begin{bmatrix} p & 5 & 2 \\ -1 & p & -8 \\ -2 & 3 & p \end{bmatrix}$$

Group - B

2. Answer *any five* questions :

(a) A company produces two types of leather belts, say type A and type B. Belt A is of a superior quality and belt B is of an inferior quality. Profits on the two types of belts are rupees 5 and rupees 4 per belt respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts are of type B, the company could produce 1000 belts per day. Belt A requires a fancy buckle and only 400 buckles are available for this per day. For belts of type B only 700 buckles are available per day. Formulate the above problem as a linear programming problem. **6**

(b) Solve graphically :

$$\text{Maximize } Z = 100x_1 + 50x_2$$
$$\text{subject to } 4x_1 + 6x_2 \leq 24,\ \ x_1 \leq 4,\ \ x_2 \leq \tfrac{4}{3},\ \ x_1, x_2 \geq 0.$$

Is there any degeneracy in the problem? Justify your answer. **3+3**

(c) Prove that x_1 and x_3 cannot be simultaneously considered as the basic variables of the set of equations :

$$2x_1 + x_2 - x_3 + 4x_4 = 6, \qquad 6x_1 + 3x_2 - 3x_3 + x_4 = 10$$

Find all the basic solutions with x_1 as one basic variable. **6**

(d) Prove that every basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. **6**

(e) Solve the following linear programming problem by simplex method :

$$\text{Minimize } Z = x_1 - 3x_2 + 2x_3$$
$$\text{subject to } 2x_1 + 3x_2 - x_3 \leq 7,\ \ 4x_3 - 2x_2 \leq 12,\ \ 8x_1 - 4x_2 + 3x_3 \leq 10,$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

6

(f) Use Charnes Big M-method to show that the following LPP has unbounded solution :

$$\text{Maximize } Z = 2x_1 + 3x_2 + x_3$$
$$\text{subject to } -3x_1 + 2x_2 + 3x_3 = 8,\ \ -3x_1 + 4x_2 + 2x_3 = 7,\ \ x_1, x_2, x_3 \geq 0$$

6

(g) Using Two-Phase Simplex method, show that the following LPP has no feasible solution :

$$\text{Maximize } Z = 5x + 3y \text{ subject to } 3x + 4y \geq 12,\ 2x + y \leq 1,\ x, y \geq 0.$$

6

(h) Show that the following LPP has unbounded solution :

$$\text{Maximize } Z = -2x_1 + 5x_2$$
$$\text{subject to } 4x_1 - 5x_2 \geq -20,\ \ x_1 + x_2 \geq 10,\ \ x_2 \geq 2,\ \ x_1, x_2 \geq 0.$$

6

Group - C

3. Answer *any five* questions :

(a) Find the dual of the following linear programming problem. Hence, prove that the dual has no optimal solution.

$$\text{Minimize } Z = 3x_1 - 5x_2 + x_3$$
$$\text{subject to } x_1 - 2x_3 \geq 4,\ \ 2x_1 - x_2 + x_3 \geq 2,\ \ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

6

(b) Suppose that a constraint in a given LPP is an equality. Prove that corresponding dual variable is unrestricted in sign.

6

(c) Solve the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	4	30
O_2	3	4	7	8	15
O_3	9	6	5	8	15
b_j	10	25	18	7	

Is the solution unique? **5+1**

(d) Solve the assignment problem with the following cost matrix : **6**

	M_1	M_2	M_3	M_4	M_5
I	9	8	7	6	4
II	5	7	5	6	8
III	8	7	6	3	5
IV	8	5	4	9	3
V	6	7	6	8	5

(e) Solve the following travelling salesman problem : **6**

From \ To	A	B	C	D
A	α	46	16	40
B	41	α	50	40
C	82	32	α	60
D	40	40	36	α

(f) Reduce the following game to 2×2 game by dominance property and then solve it. **6**

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

(g) Solve the following 4×2 game graphically : **6**

$$\text{Player A} \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

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(h) Solve the following game by linear programming technique : **6**

$$\text{A}\begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix} \quad (\text{columns : B})$$

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