UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MDC

Paper: CC-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise:
https://palmaths.com/cu-pyqs/2024/maths-mdc-sem-1-cc-1/

Write the answers to each Group in a separate answer-book.

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual meanings.

Group - A

[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$, prove that $p + \dfrac{d^2p}{d\theta^2} = \dfrac{a^2b^2}{p^3}$.

(b) Calculate the total length of the cardioide $r = a(1+\cos\theta)$.

(c) Find $\dfrac{dy}{dx}$, where $(\cos x)^y = (\sin y)^x$.

(d) Find the n-th derivative of $y = \dfrac{a-x}{a+x}$.

(e) If $I_n = \displaystyle\int_0^1 (1-x^2)^n\,dx$, then prove that $(2n+1)I_n = 2nI_{n-1}$.

(f) Find $\displaystyle\lim_{h\to 0}\frac{f(a+h)+f(a-h)-2f(a)}{h^2}$ provided $f'''(x)$ exists for all x.

(g) If $f(x) = x\sin\dfrac{1}{x}$, $x \neq 0$ and $f(0) = 0$, show that $f'(0)$ does not exist.

Answer *any three* questions :

2. (a) For all $x > 0$, prove that $\dfrac{x}{1+x} < \log(1+x) < x$. **4**

(b) Find the area in the first quadrant included between the parabola $y^2 = bx$ and the circle $x^2 + y^2 = 2bx$. Find also when $b = 4$. **4**

(c) If $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, for $|x| < 1$, show that $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2y_n = 0$. **4**

(d) If $I_n = \displaystyle\int_0^{\pi/4}\tan^n x\,dx$, show that $I_{n+1} - I_{n-1} = \dfrac{1}{n}$. *[PALMATHS note: As printed; the standard result is $I_{n+1} + I_{n-1} = \frac{1}{n}$.]* Using this relation, find the value of $\displaystyle\int_0^{\pi/4}\tan^8 x\,dx$. **2+2**

(e) Consider the straight line passing through origin O and a point $P(h,r)$. The portion OP is revolved about x-axis. Show that the surface area of the cone thus generated is πrl, where l is the slant height of the cone. Also show that the volume of the cone is $\dfrac{1}{3}\pi r^2 h$. **4**

(f) Find the value of a and b if $\lim_{x\to 0}\frac{a\sin x - bx\cos x}{x^3} = \frac{1}{3}$. **4**

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions :

(a) The coordinate axes are rotated through an angle $\frac{\pi}{4}$. If the transformed coordinates of a point are $\left(0, \sqrt{2}\right)$, find the original coordinates. $2\frac{1}{2}$

(b) Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $x+y-2z-4=0$ and the origin. $2\frac{1}{2}$

(c) Determine the nature of the conic $r = \frac{1}{4-5\cos\theta}$. Find the eccentricity and the length of the latus rectum. $\frac{1}{2}+1+1$

(d) Find the equation to the generating lines of the paraboloid $(x+y+z)(2x+y-z) = 6z$, which pass through the point $(1,1,1)$. $2\frac{1}{2}$

Answer *any five* questions : 6×5

4. (a) The tangents at two points P and Q of a parabola whose focus is S, meet at T. Show that $SP\cdot SQ = ST^2$.

(b) Reduce the equation $7x^2-6xy-y^2+4x-4y-2=0$ to its canonical form and find the nature of the conic.

(c) Find the equations of the generators of the hyperbola $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, passing through the point $(a\cos\alpha, b\sin\alpha, 0)$.

(d) If the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, then prove that $(l\cos\alpha - ep)^2+l^2\sin^2\alpha = p^2$.

(e) Find the equation of the right circular cylinder of radius 3 with axis passing through $(1,3,4)$ and have $1,-2,3$ as its direction ratios.

(f) Find the nature of the surface given by the equation $2x^2+3y^2-8x+6y-12z+11=0$.

(g) If PSP' be a focal chord of a conic, then show that $\frac{1}{SP}+\frac{1}{SP'}=\frac{2}{l}$, where l is the semi-latus rectum.

(h) Find the centre and the radius of the circle : $x^2+y^2+z^2-2x+4y+6z-2=0$, $x+3y+2z=15$.

(i) Show that the foot of the perpendicular from the focus to any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lies on the circle $x^2+y^2=a^2$.

Group - C
[Vector Analysis]
(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Prove that, $\vec{a} \times (\vec{b} \times \vec{c})$, $\vec{b} \times (\vec{c} \times \vec{a})$ and $\vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

(b) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$, show that, $\dfrac{d\vec{r}}{dt} \cdot \dfrac{d^2\vec{r}}{dt^2} \times \dfrac{d^3\vec{r}}{dt^3} = a^2 b$.

(c) Find the vector equation of the straight line passing through the point $(-1, 4, 3)$ and parallel to the vector $4\hat{i} + 3\hat{j} + 2\hat{k}$.

(d) A force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of $\vec{F}$ about the point $(2, -1, 3)$.

(e) Evaluate $\displaystyle\int_2^3 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$, where $\vec{r} = t^3\hat{i} + 2t^2\hat{j} + 3t\hat{k}$.

(f) If a particle is acted on by constant forces $(4\hat{i} + \hat{j} - 3\hat{k})$ and $(3\hat{i} + \hat{j} - \hat{k})$ and is displaced from the point $(\hat{i} + 2\hat{j} + 3\hat{k})$ to the point $(5\hat{i} + 4\hat{j} - \hat{k})$, find the total work done by the forces.

(g) Solve : $p\vec{x} + \vec{x}(\vec{x} \cdot \vec{b}) = \vec{a} \times \vec{b} + \vec{c}$.

Answer *any three* questions : 4×3

6. (a) If $\vec{F} = x^2y\hat{i} + xy^2z\hat{j} + y^3z^2\hat{k}$ and $\vec{G} = x\hat{i} - yz^2\hat{j} + xyz\hat{k}$, then show that $\dfrac{\partial^2}{\partial x \partial y}(\vec{F} \cdot \vec{G}) = \dfrac{\partial^2}{\partial y \partial x}(\vec{F} \cdot \vec{G})$ and find $\dfrac{\partial^2}{\partial x \partial y}(\phi\vec{F})$ at $(-1, -1, 1)$, where $\phi = xyz$.

(b) If $\phi = 3x^2yz$ and C is the curve $x = t^2$, $y = t^3$, $z = t$ from $t = 0$ to $t = 1$, evaluate the vector line integral $\displaystyle\int_C \phi\, d\vec{r}$.

(c) Find the vector equation of the plane passing through the point $(8\hat{i} + 2\hat{j} - 3\hat{k})$ and perpendicular to each of the planes $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 0$ and $\vec{r} \cdot (\hat{i} + 3\hat{j} - 5\hat{k}) + 5 = 0$.

(d) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of velocity and acceleration at time $t = 1$, in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$.

(e) Show that $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ are coplanar if and only if $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

(f) If $\vec{r} = (a\cos t)\hat{i} + (a\sin t)\hat{j} + (at\tan\alpha)\hat{k}$, evaluate $\left[\dfrac{d\vec{r}}{dt}\ \dfrac{d^2\vec{r}}{dt^2}\ \dfrac{d^3\vec{r}}{dt^3}\right]$.

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