UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MDC

Paper: CC-3

(Ordinary Differential Equations and Group Theory)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Ordinary Differential Equations]
(Marks : 45)

Answer *any nine* questions.

1. (a) Determine the order and degree of the differential equation $\sqrt{y+\left(\frac{dy}{dx}\right)^2}=1+x$.

(b) Obtain the differential equation of all circles each of which touches the axis of x at the origin. **2+3**

2. Solve : $(x+2y-3)dx=(2x+y-3)dy$. **5**

3. Solve : $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$. **5**

4. Solve : $x\,dx+y\,dy+\frac{x\,dy-y\,dx}{x^2+y^2}=0$. **5**

5. Find both general solution and singular solution of the ODE : $x=py-p^2$, where $p=\frac{dy}{dx}$. **5**

6. Solve : $\frac{dy}{dx}-\frac{3}{x+2}y=(x+2)^3$. **5**

7. Solve : $y+px=x^4p^2$. **5**

8. Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$. **5**

9. Solve by the method of undetermined coefficients $(D^2-3D)y=2x^2+1$. **5**

10. Solve : $(x^2D^2-3xD+5)y=x^2\sin(\log x)$. **5**

11. Solve by the method of variation of parameters, the equation $\frac{d^2y}{dx^2}+a^2y=\sec ax$. **5**

12. Show that e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. Find the solution $y(x)$ with the condition $y(0)=0$ and $y'(0)=1$. **5**

13. Solve for x from the system of equations :

$$\frac{dx}{dt}+4x+3y=t, \qquad \frac{dy}{dt}+2x+5y=e^t.$$

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5

14. Solve : $\dfrac{a\,dx}{yz(b-c)} = \dfrac{b\,dy}{zx(c-a)} = \dfrac{c\,dz}{xy(a-b)}$. **5**

15. Show that the equation $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + y = \dfrac{1}{(1-x)^2}$, $0 < x < 1$ is exact and hence solve it. **5**

16. Solve : $\dfrac{d^2y}{dx^2} = \sec^2 y \tan y$. **5**

Group - B

[Group Theory]
(Marks : 30)

Answer *any six* questions.

17. Define a group. Does the set of all 2×2 non-singular matrices over integers form a group under matrix multiplication? Justify your answer. **2+3**

18. Let $(G, \circ)$ be a semi-group and for any two elements $a, b \in G$, each of the equation $a \circ x = b$ and $y \circ a = b$ has a solution in G. Then prove that $(G, \circ)$ is a group. **5**

19. (a) If a is an element of a group G such that $o(a) = 7$, then prove that there exists an element b in G such that $b^3 = a$.

(b) Give an example of a group G and subgroups H and K such that HK fails to be a subgroup of G. Justify your claim. **2+3**

20. (a) Let $(G, *)$ be a group with identity element e. If $a^2 = e\ \forall a \in G$, show that G is commutative.

(b) In a group $(G, *)$, 'a' is an element of order 30. Find the order of a^{18}. **3+2**

21. Show that a non-empty subset H of a group $(G, \circ)$ is a subgroup of $(G, \circ)$ if and only if $\forall a, b \in H$, $a^{-1} \circ b \in H$. **5**

22. (a) Prove that the centre $Z(G)$ of a group G is a subgroup of G.

(b) Prove or disprove : $Z(S_3) = \{(1)\}$. **2+3**

23. Define cyclic group. Let $S = \{1, i, -1, -i\}$. Then show that $(S, \cdot)$ is a cyclic group. Show that every cyclic group is abelian. **1+2+2**

24. (a) Define cosets of a subgroup H in a group $(G, \circ)$.

(b) The set $H = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}$ is a subgroup of $\mathbb{Z}_{12}$. Find all cosets of H. **2+3**

25. Prove that every group of prime order is cyclic. **5**

26. If $\alpha = (1,3,4)(5,6)(2,7,8,9)$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$ be two permutations, find the product $\gamma = \beta^{-1}\alpha\beta$. Is γ an even permutation? **5**

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