UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MDC

Paper: MN-1

(Calculus, Geometry and Vector Analysis (Maths as Minor in MDC))

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Find the value of $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(b) Find $\frac{dy}{dx}$, when $y=(\sin x)^{\log x}$.

(c) If $y=(\sin^{-1}x)^2$, then show that $(1-x^2)y_2-xy_1-2=0$.

(d) If $f(x)=Kx-\sin x$ is an increasing function, then find the values of K.

(e) If $I_n=\int_0^{\pi/2}\cos^n x\,dx=\frac{n-1}{n}I_{n-2}$, then, evaluate $\int_0^{\pi/2}\cos^7 x\,dx$.

(f) Find the length of the arc of the parabola $x^2=4y$ between the points where $y=1$.

(g) Find the area bounded by the curves, $y^2=x$ and $y=x^2$.

Answer *any three* questions : 4×3

2. (a) Find a, b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$.

(b) If $y=2(\sin x-\cos x)\cos x$, show that $(y_{10})_0=2^{10}$.

(c) If $y=\tan^{-1}x$, then prove that $(1+x^2)y_{n+1}+2nxy_n+n(n-1)y_{n-1}=0$.

(d) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, show that $I_n+I_{n-2}=\frac{1}{n-1}$. Hence, find $\int_0^{\pi/4}\tan^4 x\,dx$.

(e) Find the perimeter of the curve represented by $x=\frac{1-t^2}{1+t^2}, y=\frac{2t}{1+t^2}$.

(f) Find the area of the region bounded by the curve $y=x^3$ and the line $y=2x$.

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Transform the equation $x^2+y^2-6x+4y-22=0$ on transforming the origin to $(3,-2)$.

(b) Show that the equation given by $x^2 - 2xy + 2y^2 - 4x - 6y + 3 = 0$ represents an ellipse.

(c) Find the point on the conic $\frac{14}{r} = 3 - 8\cos\theta$ whose vectorial angle is $\frac{2\pi}{3}$.

(d) Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) + 2x - 4y + 8z = 5$.

Answer *any five* questions : 6×5

4. (a) The origin is shifted to the point $(3, -1)$ and the coordinate axes are then rotated through an angle $\tan^{-1}\frac{3}{4}$. Find the coordinates of the point $(5, -2)$ in the new coordinate system.

(b) Reduce the equation $5x^2 - 6xy + 5y^2 - 4x - 4y - 4 = 0$ to its canonical form. Hence, find the nature of the curve.

(c) Show that if the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$, then $(l\cos\alpha - ep)^2 + l^2\sin^2\alpha = p^2$.

(d) If the normal to the parabola $y^2 = 4ax$ at the point $(at_1^2, 2at_1)$ meets the parabola again at $(at_2^2, 2at_2)$, then show that $t_2 = -t_1 - \frac{2}{t_1}$.

(e) If a plane meet the axes in A, B, C and centroid of $\triangle$ABC is (α, β, γ). Find the equation of the plane.

(f) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as the great circle.

(g) Find the equation of the right circular cone whose vertex is the origin and whose axis is $\frac{x}{3} = \frac{y}{1} = \frac{z}{1}$ and semi-vertical angle is $\frac{\pi}{4}$.

(h) Find the equation of the right circular cylinder whose guiding curve is the circle through the points $(a, 0, 0), (0, a, 0), (0, 0, a)$.

(i) Show that the plane $x - 2y - 2z = 18$ is a tangent plane to the ellipsoid $\frac{x^2}{144} + \frac{y^2}{36} + \frac{z^2}{9} = 1$ and find the coordinates of the point of contact.

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) For the three vectors $\vec{\alpha} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{\beta} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{\gamma} = \hat{i} - 2\hat{j} + 2\hat{k}$, find $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma}$.

(b) Find the vector equation of the line joining the points $2\hat{i} - 3\hat{j} - \hat{k}$ and $8\hat{i} - \hat{j} + 2\hat{k}$.

(c) Prove that $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$.

(d) A force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of $\vec{F}$ about the point $(2, -1, 3)$.

(e) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{b} = (2t - 3)\hat{i} + \hat{j} - t\hat{k}$, find $\frac{d}{dt}(\vec{a} \cdot \vec{b})$ at $t = 1$.

(f) If $\vec{r}(t) = (t - t^2)\hat{i} + 2t^3\hat{j} - 3\hat{k}$, then find $\int_1^2 \vec{r}\,dt$.

(g) Find the volume of the parallelopiped whose edges are represented by the vectors $\vec{\alpha} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$, and $\vec{\gamma} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Answer *any three* questions : 4×3

6. (a) Prove that $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\gamma} \times \vec{\delta}) = (\vec{\alpha} \cdot \vec{\gamma})(\vec{\beta} \cdot \vec{\delta}) - (\vec{\alpha} \cdot \vec{\delta})(\vec{\beta} \cdot \vec{\gamma})$.

(b) Write down vector equation of a plane through P, Q and R having position vector $2\hat{i} + 2\hat{j} - \hat{k}$, $3\hat{i} + 4\hat{j} + 2\hat{k}$ and $7\hat{i} + 6\hat{k}$ respectively.

(c) A particle, acted on by constant forces $4\hat{i} + 2\hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j}$ to $2\hat{i} - \hat{j} + 3\hat{k}$. Find the work done by the forces.

(d) If $\vec{r} = 5\cos t\,\hat{i} + 5\sin t\,\hat{j} + 7t\,\hat{k}$, then prove that $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right] = 175$.

(e) If $\vec{\alpha} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and $\vec{\beta} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$, then find $\frac{d}{dt}\left(\vec{\alpha} \times \frac{d\vec{\beta}}{dt}\right)$ at $t = 2$.

(f) If $\vec{F}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, then find $\int_1^2 \vec{F}(t) \times \frac{d^2}{dt^2}\vec{F}(t)\,dt$.

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