UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MINOR

Paper: MN-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Find the value of $\lim_{x \to 0} \dfrac{\tan x - x}{x - \sin x}$.

(b) Find $\dfrac{dy}{dx}$, when $x = a\cos^3 t$ and $y = b\sin^3 t$.

(c) Find the values of k in order that $f(x) = (k^2 - 5k + 18)x + x^3 + 6x^2$ is a decreasing function.

(d) If $y = (\sin^{-1} x)^2$, then show that $(1 - x^2)y_2 - xy_1 - 2 = 0$.

(e) Evaluate $\int_0^{\pi/2} \sin^7 x\, dx$ using the reduction formula, $I_n = \int_0^{\pi/2} \sin^n x\, dx = \dfrac{n-1}{n} I_{n-2}$.

(f) Find the arc length of the curve, $x = \cos 3t$, $y = \sin 3t$ over the interval $t \in [0, \pi]$.

(g) Find the area in the first quadrant included between the parabola $x^2 = 36y$, the Y-axis and the line $y = 4$.

Answer *any three* questions : 4×3

2. (a) Find the values of a and b in order that $\lim_{x \to 0} \dfrac{a\sin 2x - b\sin x}{x^3} = 1$.

(b) Find $\dfrac{dy}{dx}$ when $\tan^{-1}\dfrac{y}{x} = \log(x^2 + y^2)$.

(c) State Leibnitz Theorem on successive differentiation. If $y = e^{-x}\cos x$, then prove that $y_4 + 4y = 0$.

(d) Find the area enclosed by the curve, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

(e) Find the length of the arc of the cardioide $r = a(1 + \cos\theta)$ from the vectorial angle 0 to θ.

(f) If $y = 2\cos x(\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$.

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

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(a) Find the angle through which the axes be turned to remove the term xy from the equation, $ax^2 + 2hxy + by^2 = 0$.

(b) Determine the nature of the conic, $\frac{3}{r} = 2 + 4\cos\theta$ and also find the length of its latus rectum.

(c) Find the equation of the sphere whose extremities of a diameter are $(3, 4, -2)$ and $(-1, 3, 2)$.

(d) Find the value of k for which the equation, $kx^2 + 8xy + 4y^2 + 6x + 4y + 1 = 0$ represents a conic without any centre.

Answer *any five* questions : 6×5

4. (a) Reduce the equation, $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ to its canonical form and determine the nature of the conic.

(b) If the straight line, $\frac{1}{r} = A\cos\theta + B\sin\theta$ touches the conic, $\frac{l}{r} = 1 + e\cos\theta$, then show that $(lA - e)^2 + l^2B^2 = 1$.

(c) If the normal to the hyperbola, $xy = c^2$ at the point $\left(ct_1, \frac{c}{t_1}\right)$ meet it again at $\left(ct_2, \frac{c}{t_2}\right)$, then show that $t_1^3 t_2 + 1 = 0$.

(d) Find the equation of the locus of the points of intersection of mutually perpendicular tangents to the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(e) A sphere of radius r passes through the origin $(0, 0)$ and touches the axes in P, Q, R. Find the locus of the centroid of the triangle PQR.

(f) Find the equation of the cone with vertex $(1, 1, 3)$ and guiding curve $x^2 + 2y^2 = 1, z = 4$.

(g) Find the equation of the cylinder whose generators are parallel to the line, $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and the guiding curve is the ellipse, $x^2 + 4y^2 = 9, z = 1$.

(h) Find the equations of the generating lines of the paraboloid $4x^2 - y^2 = 8z$ passing through the point $(-3, 2, 4)$.

(i) Show that the quadric surface given by the equation $3x^2 + 5y^2 + 3z^2 + 2yz + 2zx + 2xy - 4x - 8z + 5 = 0$ is a central conicoid. Find its centre.

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) If $\vec{\alpha} = \hat{i} + \hat{j} - \hat{k}, \vec{\beta} = 3\hat{i} - \hat{k}, \vec{\gamma} = 2\hat{i} - 3\hat{j}$, find $\left[\vec{\alpha}, \vec{\beta}, \vec{\gamma}\right]$.

(b) Prove that $\left[\vec{a} + \vec{b} \;\; \vec{b} + \vec{c} \;\; \vec{c} + \vec{a}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

(c) Find the vector equation of the line passing through the point, $-2\hat{i} + 3\hat{j} + 4\hat{k}$ and parallel to the vector, $2\hat{i} + 3\hat{j} + 4\hat{k}$.

(d) Find the values of a and b for which the vectors $8\hat{i} - 3\hat{j} + a\hat{k}$ and $4\hat{i} + b\hat{j} + 4\hat{k}$ are collinear.

(e) Find the work-done in moving an object along a vector $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$.

(f) Find the area of the triangle having vertices at $\hat{i} + 3\hat{j} + 2\hat{k}, 2\hat{i} - \hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$.

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(g) If $\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j} + t\hat{k}$, then find the value of $\left|\dfrac{d^2\vec{r}}{dt^2}\right|$.

Answer *any three* questions : 4×3

6. (a) Show by vector method that the line joining the middle points of two sides of a triangle is parallel to the third side and half of its length.

(b) Find the vector equation of a plane through the three points $(-2, 6, -6)$; $(-3, 10, -9)$ and $(-5, 0, -6)$.

(c) $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$ are three non-coplanar vectors, then show that $(\vec{\alpha} - \vec{\beta}) \cdot (\vec{\beta} - \vec{\gamma}) \times (\vec{\gamma} - \vec{\alpha}) = -2\left[\vec{\alpha}, \vec{\beta}, \vec{\gamma}\right]$. *[PALMATHS note: As printed; this scalar triple product is actually 0 for any three vectors.]*

(d) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of velocity and acceleration at time $t = 1$.

(e) Find the torque about the point $B(3, -1, 3)$ of a force $\vec{P}(4, 2, 1)$ passing through the point $A(5, 2, 4)$. Find the magnitude of the torque.

(f) If $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$, then find $\displaystyle\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$.

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