UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MINOR

Paper: MN-2

(Basic Algebra)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Throughout the question paper $\mathbb{R}$ denotes the set of real numbers.
Other symbols have their usual meanings.

Group - A
(Marks : 25)

1. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the cube roots of -1.

(b) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^8 - 1 = 0$.

(c) Show that $(n+1)^n > 2^n \cdot n!$, where n is any positive integer.

(d) If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P., then show that $p^2 \geq 3q$.

Answer *any four* questions : 5×4

2. (a) Prove that $\sin\left[i \log \frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero.

(b) Solve the equation $x^3 - 18x - 35 = 0$, by Cardan's method.

(c) Solve the equation $2x^3 - x^2 - 18x + 9 = 0$ if two of the roots are equal in magnitude but opposite in sign.

(d) Using Cauchy-Schwartz inequality, prove that $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) > 16$, where a, b, c, d are all positive real numbers (not all equal).

(e) Solve the equation $\cos z = 2$, where the solutions should be written in general form.

(f) a, b, c be positive real numbers, prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} > \frac{3}{2}$, unless $a = b = c$.

(g) Find the sum of 99th powers of the roots of the equation $x^7 - 1 = 0$.

Group - B
(Marks : 25)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Suppose $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 5x + 6$. Prove that f is neither an injective mapping nor a surjective mapping.

(b) Find two integers u and v such that $54u + 24v = 30$.

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(c) Find out the unit digit of 3^{2024}.

(d) Examine whether the relation R, defined on the set $\mathbb{Z}$ by $a\,R\,b$ if and only if $a-b<5$, $a,b\in\mathbb{Z}$ is an equivalence relation or not.

Answer *any four* questions :

4. (a) Find gcd of 315 and 4235 and find integers s and t such that $\gcd(315,4235)=315s+4235t$. **5**

(b) Find the remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\cdots+\lfloor 100$ is divided by 40. (Here $\lfloor n$ denotes $n!$.) **5**

(c) Find $f\circ g$ and $g\circ f$, where $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=|x|+x$, $x\in\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ is defined by $g(x)=|x|-x$, $x\in\mathbb{R}$. **5**

(d) Solve the system of linear congruences by Chinese remainder theorem

$$x\equiv 2 \pmod 3,\quad x\equiv 3 \pmod 5,\quad x\equiv 4 \pmod 7$$

5

(e) (i) Find the value of $\phi(2024)$.

(ii) n^2+n+41 is a prime number, for all $n\in\mathbb{N}$. Is it true? Justify your answer. **3+2**

(f) If $A=\{2,3,4\}$, then find the number of relations on A which are both reflexive and symmetric.

5

(g) Let $A=\mathbb{R}\setminus\left\{-\frac{1}{2}\right\}$, $B=\mathbb{R}\setminus\left\{\frac{1}{2}\right\}$, where $\mathbb{R}$ denotes the set of all real. Let $f:A\to B$ be defined by $f(x)=\dfrac{x-3}{2x+1}$, for all $x\in A$. Does f^{-1} exist? Justify your answer. **1+4**

Group - C
(Marks : 25)

5. Answer *any two* questions : $2\frac{1}{2}\times 2$

(a) Verify whether the set of vectors $\{(1,5,2),(1,1,0),(0,0,1)\}$ is linearly independent or not in $\mathbb{R}^3$.

(b) Find a spanning set of the subset S of $\mathbb{R}^3$, where $S=\{(x,y,z)\in\mathbb{R}^3 : 2x+y-z=0\}$.

(c) If A is invertible and $AB=0$, then prove that $B=0$, where A, B both $n\times n$ (square) matrices.

(d) Find the value of λ so that the matrix $A=\begin{pmatrix}1 & 0\\ 1 & \lambda\end{pmatrix}$ is its own inverse.

Answer *any four* questions : 5×4

6. (a) Find the rank of the matrix by reducing it to row reduced Echelon form

$$\begin{pmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & 2 & 0\end{pmatrix}.$$

(b) Determine the conditions for which the system $x+y+z=1$, $x+2y-z=b$, $5x+7y+az=b^2$ admits of (i) only one solution, (ii) no solution, (iii) many solutions.

(c) Prove that $(A+B)A^{-1}(A-B) = (A-B)A^{-1}(A+B)$ if A and B are square matrices and A is invertible.

(d) If $\{\alpha, \beta, \gamma\}$ is linearly independent set of vectors in space $\mathbb{R}^n$, then prove that $\{\alpha+\beta+\gamma,\ \beta+\gamma,\ \gamma\}$ is also a linearly independent set in $\mathbb{R}^n$.

(e) Find the dimension and basis of the solution space of the system of equations :

$$x - 2y + z = 0, \qquad x - 2y - z = 0$$

(f) Transfer the system of equation $2x - 3y + 4z = 3, 3x + 2y - z = 4, 5x + 3y - z = 7$ to a matrix equation $AX = B$. Hence, solve the system by using the properties of matrix.

(g) Find a linearly independent subset T of the set $S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$, where $\alpha_1 = (1,2,1)$, $\alpha_2 = (-3,-6,3)$, $\alpha_3 = (2,1,3)$, $\alpha_4 = (8,7,7) \in \mathbb{R}^3$ which spans the same space as S.

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