UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2024

MATHEMATICS — MINOR

Paper: MN-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Find $f'(0)$, if it exists, for $f(x) = \begin{cases} 3+2x, & -\frac{3}{2} < x \leq 0 \\ 3-2x, & 0 \leq x < \frac{3}{2} \end{cases}$

(b) If $y = \tan^{-1} x$, then show that, $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

(c) Evaluate the limit : $\lim\limits_{x \to 1} \left[\dfrac{x}{x-1} - \dfrac{1}{\log_e x} \right]$

(d) Using the reduction formula for $I_n = \displaystyle\int_0^{\pi/2} \sin^n \theta \, d\theta = \frac{n-1}{n} I_{n-2}$, evaluate $\displaystyle\int_0^1 \frac{x^4}{\sqrt{1-x^2}} \, dx$.

(e) Find the length of the circumference of the circle $x^2 + y^2 = 25$.

(f) Find the area of the region bounded by the curve $y = x^3$ and the line $y = 2x$.

(g) Find the surface area generated by revolving the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 1$, $x = 4$ about x-axis.

Answer *any three* questions :

2. (a) If $y = (\sin^{-1} x)^2$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. **4**

(b) Evaluate $\lim\limits_{x \to +\infty} \left(\dfrac{x^3}{x^2 - x + 1} - \dfrac{x^3}{x^2 + x - 1} \right)$. **4**

(c) Find the reduction formula for $\displaystyle\int_0^{\pi/2} \cos^n x \, dx$, where n is a positive integer, hence, find the value of $\displaystyle\int_0^{\pi/2} \cos^4 x \sin^2 x \, dx$. **2+2**

(d) Find the exact arc length of the curve given by the equations : $x = a\cos^3\phi$, $y = a\sin^3\phi$. **4**

(e) Find the volume of the solid of revolution generated by the region enclosed by $y = 5x - x^2$, y-axis and $x = 5$ about x-axis. **4**

(f) Derive the reduction formula for $\int x^n e^{ax} dx$ and hence, evaluate $\int x^3 e^{2x} dx$. **4**

Group - B
[Geometry]

(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Obtain the parallel displacement of the axes by which the equation $x^2 + y^2 - 4x + 8y - 17 = 0$ is transformed to $x'^2 + y'^2 = 37$.

(b) Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$.

(c) Find the polar coordinates of the centre and radius of the circle : $r = 8\sin(\theta - \pi/3)$.

(d) Find the value of 'a' for which the plane $x + y + z = \sqrt{3}a$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$.

Answer *any five* questions :

4. (a) Reduce the equation $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$ to its canonical form and hence, determine the nature of the conic. **6**

(b) A straight line touches both $x^2 + y^2 = 2a^2$ and $y^2 = 8ax$. Find its equation. **6**

(c) If PSP' and QSQ' be two perpendicular focal chords of a conic with focus S, then prove that $\left(\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}\right)$ is constant. **6**

(d) Find the equation of the sphere which passes through the points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and which touches the plane $2x + 2y - z = 15$. **6**

(e) The radius of a right circular cylinder is 5, axis passes through the point $(1,2,3)$ and is parallel to the straight line $\frac{x-4}{2} = \frac{y-3}{-1} = \frac{z-2}{2}$; find the equation of the cylinder. **6**

(f) Find the nature of the surface represented by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$. Find also, the coordinates of the centre and equations of principal planes. **3+1+2**

(g) Show that the equation of the cone which passes through the coordinate axes and the straight lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{3} = \frac{y}{2} = \frac{z}{-1}$ is $3yz + 10zx + 6xy = 0$. **6**

(h) Find the equations of the generators of the hyperboloid $x^2 - y^2 = 2z$ which passes through the point $(5,3,8)$. **6**

(i) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 9$, $z = 1$. **6**

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Determine all possible values of λ for which the vector $\vec{\alpha} = \lambda(2\hat{i} - 2\hat{j} + 6\hat{k})$ is a unit vector.

(b) If $|\vec{\alpha}| = 4$, $|\vec{\beta}| = 5$ and $\vec{\alpha} \cdot \vec{\beta} = 0$, find $|\vec{\alpha} \times \vec{\beta}|$.

(c) Find, by vector method, the volume of the tetrahedron whose vertices are $(1,1,0)$, $(1,0,1)$, $(0,1,1)$ and $(1,1,1)$.

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(d) Find the value of p so that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar.

(e) A particle being acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $(1, 2, 3)$ to the point $(5, 4, -1)$. Find the total work done.

(f) A force $5\hat{i} + 2\hat{j} - 3\hat{k}$ is applied at the point $(1, -2, 2)$. Find the value of the moment of the force about the origin.

(g) If $\vec{F} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{G} = \sin t\,\hat{i} - \cos t\,\hat{j}$, find $\frac{d}{dt}\left(\vec{F} \times \vec{G}\right)$.

Answer *any three* questions :

6. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. **4**

(b) Prove by vector method that the diagonals of a rhombus are perpendicular to each other. **4**

(c) Prove that a necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$. **4**

(d) (i) If $\vec{r} = (\cos nt)\hat{i} + (\sin nt)\hat{j}$, where n is a constant and t varies, then show that $\vec{r} \times \frac{d\vec{r}}{dt} = n\hat{k}$.

(ii) If $\vec{u} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{v} = (2t - 3)\hat{i} + \hat{j} - t\hat{k}$, then show that $\frac{d}{dt}(\vec{u} \times \vec{v}) = 7\hat{j} + 3\hat{k}$ at $t = 1$. **2+2**

(e) Derive the vector equation of the straight line passing through the points $(\hat{i} - 2\hat{j} + \hat{k})$ and $(3\hat{k} - 2\hat{j})$. **4**

(f) Evaluate $\int_2^3 \left(\vec{r} \times \frac{d^2\vec{r}(t)}{dt^2}\right) dt$, where $\vec{r}(t) = t^3\hat{i} + 2t^2\hat{j} + 3t\hat{k}$. **4**

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