UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — HONOURS

Paper: DSCC-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any two* questions : 2×2

(a) If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, show that $I_n + I_{n-2} = \dfrac{1}{n-1}$.

(b) Find the length of the curve $y = \log \sec x$ between $x = 0$ and $x = \dfrac{\pi}{3}$.

(c) Evaluate $\lim\limits_{x \to 0} \left(\dfrac{\tan x}{x} \right)^{\frac{1}{x^2}}$.

Answer *any four* questions :

2. (a) Determine the value of p and q for which $\lim\limits_{x \to 0} \dfrac{x(1 + p\cos x) - q \sin x}{x^3}$ exists and equal to 1. **2+2**

(b) If $y = \cosh(\sin^{-1} x)$, then show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+1)y_n = 0$. Hence find the value of y_n at $x = 0$. **3+1**

(c) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$ and $n > 1$, prove that $I_n + n(n-1)I_{n-2} = n\left(\dfrac{\pi}{2}\right)^{n-1}$. Hence find $\int_0^{\pi/2} x^4 \sin x \, dx$. **3+1**

(d) Determine the length of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, measured from the origin to any point $\theta = \theta_1$. **4**

(e) Find the area in the first quadrant included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. **4**

(f) Find the area of the surface generated by revolving the parabola $y^2 = 4ax$ about x-axis bounded by the section $x = a$. **4**

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the point on the conic $r = \dfrac{21}{5 - 2\cos\theta}$, which has the smallest radius vector.

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(b) Find the condition for which the straight line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(c) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9, x + y + z + 3 = 0$ as a great circle.

Answer *any five* questions :

4. (a) Reduce the equation $x^2 - 4xy + 4y^2 + 2x - 4y + c = 0$ to its canonical form and determine the type of the conic represented by it for different values of c. **6**

(b) Prove that the locus of the point of intersection of two perpendicular tangents to the conic $\frac{l}{r} = 1 + e\cos\theta$, is $(e^2 - 1)r^2 - 2elr\cos\theta + 2l^2 = 0$. **6**

(c) Show that the locus of the middle points of the normal chords of the parabola $y^2 = 4ax$ is $\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$. **6**

(d) Prove that the centres of the spheres which touch the lines $y = mx, z = c$ and $y = -mx, z = -c$ lie upon the conicoid $mxy + c(1 + m^2)z = 0$. **6**

(e) Show that the plane $ax + by + cz = 0$ may cut the cone $xy + yz + zx = 0$ in perpendicular lines if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$. **6**

(f) Find the locus of the luminous point if the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ cuts circular shadow on the plane $z = 0$. **6**

(g) Find the equations of the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point $(1, 1, 1)$. Hence find the angle between these generators. **5+1**

(h) Show that the normals to the ellipsoid $x^2 + 2y^2 + 3z^2 = 50$ at the points $(6, 1, 2)$ and $(6, -1, 2)$ lie in the plane $x - z = 4$. **6**

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any two* questions : 2×2

(a) Prove the identity $\left[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$.

(b) If $\vec{\alpha} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{\beta} = (2t - 3)\hat{i} + \hat{j} - t\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meanings, then find $\left\{\vec{\alpha} \times \frac{d\vec{\beta}}{dt}\right\}$ at $t = 2$.

(c) If $\vec{r} = t^2\hat{i} - e^{\sin\left(\frac{t\pi}{2}\right)}$ for $t \in [-1, 1]$, then find $\int_{-1}^{1} \left(\vec{r} \cdot \frac{d\vec{r}}{dt}\right) dt$.

Answer *any four* questions :

6. (a) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}\vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$, are non-parallel. **4**

(b) Find the vector equation of the plane passing through the points $(1, 1, 1)$ and $(2, -1, 3)$ and parallel to the vector $3\hat{i} - 4\hat{j} + 4\hat{k}$. **4**

(c) Show that the perpendicular distance of any point $\vec{a}$ from the line $\vec{r} = \vec{b} + \lambda\vec{c}$ is $\dfrac{\left|(\vec{b}-\vec{a})\times\vec{c}\right|}{|\vec{c}|}$. **4**

(d) If $\vec{r}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, then evaluate

(i) $\displaystyle\int_1^2 \left(\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right)dt$

(ii) $\displaystyle\int_1^2 \left(\frac{1}{r}\frac{d\vec{r}}{dt} - \frac{dr}{dt}\frac{\vec{r}}{r^2}\right)dt$, where $r = |\vec{r}|$. **2+2**

(e) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\,\hat{k}$, where $\hat{i},\hat{j},\hat{k}$ have their usual meanings, find the value of $\left|\dfrac{d\vec{r}}{dt}\times\dfrac{d^2\vec{r}}{dt^2}\right|$. **4**

(f) For any two vectors $\vec{u}$ and $\vec{v}$, prove that $\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right) = (1-\vec{u}\cdot\vec{v})^2 + |\vec{u}+\vec{v}+(\vec{u}\times\vec{v})|^2$. **4**

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