UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — HONOURS

Paper: DSCC-3

(Real Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
$\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$ *denote the set of all natural, real and rational numbers respectively.*

Group - A
(Marks : 30)

1. Answer *any three* questions :

(a) Show that the set $\left\{x \in \mathbb{R} : \sin\frac{1}{x} = 0\right\}$ is countable. **3**

(b) Give example of a countable set whose derived set is uncountable. **3**

(c) Prove or disprove : If each point of a set S is isolated then S cannot have limit points. **3**

(d) Find the isolated points of the set $\left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$. **3**

(e) What is limit point of a set? Find all limit points of the set $\left\{\frac{2}{x+1} : x \in (-1,1)\right\}$ in $\mathbb{R}$. **1+2**

Answer *any three* questions :

2. (a) (i) Let A and B be bounded non-empty subset of $\mathbb{R}$ and let $A - B = \{a - b : a \in A,\ b \in B\}$. Prove that $\inf(A - B) = \inf A - \sup B$.

(ii) Show that $(0,1)$ is not enumerable. **3+4**

(b) (i) Find $\sup A$ and $\inf A$ where $A = \{x \in \mathbb{R} : 3x^2 + 8x - 3 < 0\}$.

(ii) Examine whether the set $\{x \in \mathbb{R} : 1 < |x - 2| \leq 2\}$ is open or closed. **3+4**

(c) Prove or disprove :

(i) Every non-empty bounded subset of $\mathbb{Q}$ has a lub in $\mathbb{Q}$.

(ii) $(A_1 \cap A_2)^d = A_1^d \cap A_2^d$ for any two subsets A_1 and A_2 of $\mathbb{R}$, where A^d denotes the derived set of A. **4+3**

(d) (i) Prove that each interior point of a set is a limit point. Is the converse true?

(ii) Prove that the union of a finite number of closed sets in $\mathbb{R}$ is closed. Does the conclusion hold good for arbitrary union of closed sets? Justify. **(2+1)+(2+2)**

(e) (i) Prove or disprove : The union of an infinite number of closed sets in $\mathbb{R}$ is always a closed set.

(ii) Define closed set and prove that the intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is a closed set. **3+(1+3)**

Group - B

(Marks : 35)

3. Answer *any four* questions : 2×4

(a) Prove or disprove : $\left\{\left(\frac{4}{5}\right)^n + \left(\frac{5}{4}\right)^n\right\}$ is a convergent sequence.

(b) Prove or disprove : If $\lim_{n\to\infty}(x_{n+1} - x_n) = 0$, then $\{x_n\}$ is a Cauchy sequence.

(c) Prove or disprove : If $\lim_{n\to\infty} x_n = 0$, then $\lim_{n\to\infty} x_n y_n = 0$ for every sequence $\{y_n\}$.

(d) Find $\limsup x_n$, where $x_n = \left(1 - \frac{1}{n^2}\right)\sin\frac{n\pi}{2}$, $n \in \mathbb{N}$.

(e) Find $\lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}}$.

(f) Prove or disprove : Every bounded sequence is a Cauchy sequence.

Answer *any three* questions :

4. (a) (i) Give example of two non-convergent sequences $\{x_n\}$ and $\{y_n\}$ such that $\{x_n y_n\}$ is convergent.

(ii) Let $\{x_n\}$, $\{y_n\}$ be two convergent sequences such that $x_n \le y_n$ for all $n \ge 2026$. Prove that $\lim_{n\to\infty} x_n \le \lim_{n\to\infty} y_n$. **2+2**

(b) Prove that every convergent sequence is bounded. Is the converse true? Justify. **2+2**

(c) Let $\{x_n\}$ and $\{y_n\}$ be two bounded sequences of positive real numbers. Prove that $\limsup(x_n y_n) \le \limsup x_n \cdot \limsup y_n$. Cite an example for which the strict inequality holds. **3+1**

(d) (i) Show that $\lim_{n\to\infty} x^n = 0$ if $|x| < 1$.

(ii) Let $\{u_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$. If $0 \le l < 1$, then prove that $\lim_{n\to\infty} u_n = 0$. **2+2**

(e) Show that a Cauchy sequence of real numbers is convergent. **4**

5. Answer *any three* questions :

(a) Prove that every bounded sequence has a convergent subsequence. **5**

(b) (i) If $u_n > 0$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l\ (\neq 0)$ then prove that $\lim_{n\to\infty} \sqrt[n]{u_n} = l$.

(ii) Prove that $\lim_{n\to\infty} \frac{(n!)^{1/n}}{n} = \frac{1}{e}$. **3+2**

(c) A sequence $\{x_n\}$ is defined by $x_{n+2} = \frac{1}{2}(x_{n+1} + x_n)$ for $n \in \mathbb{N}$ and $x_1 > x_2 > 0$. Show that the subsequence $\{x_{2n-1}\}$ is decreasing and the subsequence $\{x_{2n}\}$ is increasing, both are convergent and converge to the same limit $\frac{x_1 + 2x_2}{3}$. **5**

(d) (i) Show that a bounded sequence $\{u_n\}$ is convergent if and only if $\limsup u_n = \liminf u_n$.

(ii) Prove that $\lim_{n\to\infty} \frac{\{(n+1)(n+2)\cdots(2n)\}^{\frac{1}{n}}}{n} = \frac{4}{e}$ using Cauchy's second limit theorem. **3+2**

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(e) State and prove the sandwich theorem and by using this theorem prove that

$$\lim_{n\to\infty} \frac{3n^3+2n^2+1}{n^3+1} = 3.$$

1+2+2

Group - C

(Marks : 10)

6. Answer *any two* questions :

(a) State Gauss' Test. Examine the convergence of the series $\left(\frac{1}{2}\right)^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 + \ldots$ **1+4**

(b) Prove or disprove :

(i) If a series $\sum x_n$ is convergent, then $\sum x_n^2$ is also convergent.

(ii) If $\sum y_n^2$ is convergent, then $\sum y_n$ is also convergent. **2+3**

(c) (i) State Raabe's test.

(ii) Show that the series $1 + \frac{1}{2}\cdot\frac{1}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{7} + \ldots$ is convergent. **2+3**

(d) State Leibniz's test. Prove that the series $\frac{3}{1\cdot 2} - \frac{5}{2\cdot 3} + \frac{7}{3\cdot 4} - \ldots$ converges conditionally. **1+4**

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