UNIVERSITY OF CALCUTTA · Previous Year Question Paper

# 2025

## MATHEMATICS — HONOURS

**Paper: DSCC-4**

**(Ordinary Differential Equations - I and Group Theory - I)**

**Full Marks: 75**

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*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

### Group - A

(Ordinary Differential Equations - I)

Answer *any nine* questions.

**1.** Find the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$. **5**

**2.** Show that $\frac{1}{3x^3y^3}$ is an integrating factor of $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ and hence solve it. **2+3**

**3.** Find the value of constant $\lambda$ for which the differential equation $(2xe^y + 3y^2)dy + (3x^2 + \lambda e^y)dx = 0$ is exact. Hence solve the equation. **2+3**

**4.** Solve : $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. **5**

**5.** Reduce the differential equation $y = 2px - p^2y$ to Clairaut's form by the substitutions $y^2 = Y$, $x = X$ and then obtain the complete primitive and singular solution, if any. **5**

**6.** Solve $(D^3 + 3D^2 - 4)y = xe^{-2x}$ $\left(D \equiv \frac{d}{dx}\right)$. **5**

**7.** Solve the following differential equation by the method of variation of parameters :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$$

**5**

**8.** Solve the differential equation $(D^2 - 2D + 5)y = e^x \sin 2x$ by the method of undetermined coefficients.

**5**

**9.** Solve : $\left(x^2D^2 - xD + 4\right) y = \cos(\log x) + x \sin(\log x)$. **5**

**10.** Solve the following by changing the independent variables :

$$\left(1 + x^2\right)^2 \frac{d^2y}{dx^2} + 2x \left(1 + x^2\right) \frac{dy}{dx} + 4y = 0.$$

**5**

**11.** Solve the following differential equation by factorisation of operators :

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x.$$

**5**

**12.** Solve the differential equation $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = e^x$, by showing that it is exact. **5**

**13.** Solve : $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = y^2 \log y$. **5**

**14.** Solve for $x$ and $y$ :

$$\frac{dx}{dt} - 3x + 4y = e^{-2t}, \qquad \frac{dy}{dt} - x + 2y = 3e^{-2t}$$

**5**

## Group - B

(Group Theory - I)

Answer *any six* questions.

**15.** (a) In $S_8$, Let $\alpha = (1\,2\,3\,4\,5)$ and $\beta = (5\,6\,7)$. Find the order of the permutation $\alpha\beta^{-1}$.

(b) Find the image of the elements 3 and 4 if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & & & 4 \end{pmatrix}$ is an odd permutation. **3+2**

**16.** Let $(G, \circ)$ be a semi-group and for $a, b \in G$, the equations $a \circ x = b$ and $y \circ a = b$ has a solution in $G$. Then prove that $(G, \circ)$ is a group. **5**

**17.** Prove that the set of matrices $M$ given by, $M = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} ; x \in \mathbb{R}, x \neq 0 \right\}$ forms an abelian group under matrix multiplication. **5**

**18.** (a) Prove that a group $G$ is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$, for all $a, b \in G$.

(b) State whether the set $G = \{a^m b^n : m, n \in \mathbb{Z}\}$ is a group under multiplication, where $a, b > 0$ are fixed real numbers. Justify your answer. **3+2**

**19.** In a group $(G, \circ)$, the elements $a$ and $b$ commute and $o(a)$ and $o(b)$ are prime to each other. Show that $o(a \circ b) = o(a) \cdot o(b)$. **5**

**20.** Prove that every cyclic group is an abelian group. Is the converse true? Justify your answer. **3+2**

**21.** (a) Prove that centre of a group is a subgroup of the group.

(b) Find all elements of order 5 in the group $(\mathbb{Z}_{20}, +)$. **2+3**

**22.** State and prove Lagrange's theorem. **1+4**

**23.** Prove that every group of order less than 6 is commutative. **5**

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