UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — HONOURS

Paper: SEC-3

(Linear Programming and Rectangular Games)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any five* questions : 3×5

(a) Solve the following LPP by graphical method :

$$\text{Minimize } Z = 4x_1 + x_2$$
$$\text{subject to } x_1 + 2x_2 \le 3,\ \ 4x_1 + 3x_2 = 6,\ \ 3x_1 + x_2 \ge 3,\ \ x_1, x_2 \ge 0$$

(b) Find a basic solution of

$$x_1 + 4x_2 - x_3 = 3, \qquad 5x_1 + 2x_2 + 3x_3 = 4$$

with x_3 as non-basic variable.

(c) Prove that a hyperplane is a convex set.

(d) Prove that the set $X = \{(x, y) \in E^2 : y^2 \le x\}$ is a convex set.

(e) Using simplex method, solve the following LPP :

$$\text{Maximize } Z = 3x_1 + 9x_2$$
$$\text{subject to } 2x_1 + 3x_2 \le 6,\ \ 2x_1 + 6x_2 \le 6,\ \ x_1, x_2 \ge 0.$$

(f) Find the dual of the following LPP :

$$\text{Maximize } Z = x_1 - x_2 + 3x_3 + 2x_4$$
$$\text{subject to } x_1 + x_2 \ge -1,\ \ x_1 - 3x_2 - x_3 \le 7,\ \ x_1 + x_3 - 3x_4 = -2,$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

(g) Obtain an intial basic feasible solution to the following Transportation problem using the matrix minima method :

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	

(h) Apply maximin and minimax principle to solve the game whose pay-off matrix is given below :

Player A \ Player B	B_1	B_2	B_3
A_1	-1	-2	8
A_2	7	5	-1
A_3	6	0	12

Group - B

2. Answer *any five* questions :

(a) A furniture manufacturer plans to make only two products, chairs and tables, from his available resources. He has 400 board feet of wood and 450 man-hours. A chair requires 5 board feet of wood and 10 man-hours and yields a profit of Rs. 140.00, while one table requires 20 board feet of wood and 15 man-hours and yields a profit of Rs. 300. Pose a linear programming problem so that he can maximize his profit. **6**

(b) (i) Is $(2, 0, 0, 1, 0)$ a basic feasible solution of the system of equations

$$2x_1 + 2x_2 - x_3 + x_4 + 5x_5 = 5, \qquad 4x_1 + 3x_2 - 3x_3 + 2x_4 - 3x_5 = 10?$$

Justify your answer.

(ii) $(2, 4, 5)$ is a feasible solution of the set of equations

$$2x_1 - x_2 + 2x_3 = 10, \qquad x_1 + 4x_2 = 18.$$

Reduce it to a basic feasible solution of the set. **2+4**

(c) Prove that the objective function of a linear programming problem attains its optimal value (if it exists) at an extreme point of the convex set of feasible solutions. **6**

(d) What do you mean by degenerate basic feasible solution? Solve graphically to show that the LPP

$$\text{Maximize } Z = 3x_1 + 9x_2 \text{ subject to } x_1 + 4x_2 \le 8,\ x_1 + 2x_2 \le 4,\ x_1, x_2 \ge 0$$

admits of a degenerate optimal basic feasible solution. **2+4**

(e) Solve the following linear programming problem by simplex method :

$$\text{Maximize } Z = 3x_1 + 5x_2 + 2x_3$$
$$\text{subject to } x_1 + x_2 + 2x_3 \le 430,\ \ 3x_1 + x_2 \le 460,\ \ x_1 + 4x_3 \le 420,\ \ x_1, x_2, x_3 \ge 0.$$

6

(f) Use Big M method to solve the following LPP :

$$\text{Maximize } Z = 2x_1 + x_2 + 3x_3$$
$$\text{subject to } x_1 + x_2 + 2x_3 \le 5,\ \ 2x_1 + 3x_2 + 4x_3 = 12,\ \ x_1, x_2, x_3 \ge 0.$$

6

(g) Solve the following LPP by two-phase method :

$$\text{Minimize } Z = 4x_1 + x_2$$
$$\text{subject to } 3x_1 + x_2 = 3,\ \ 4x_1 + 3x_2 \ge 6,\ \ x_1 + 2x_2 \le 4,\ \ x_1, x_2 \ge 0.$$

6

(h) (i) Show that the following LPP has no feasible solution :

Maximize $Z = 3x_1 + 2x_2$ subject to $2x_1 + x_2 \leq 2,\ 3x_1 + 4x_2 \geq 12,\ x_1, x_2 \geq 0.$

(ii) Prove that the set of all feasible solutions of a linear programming problem is a convex set. **3+3**

Group - C

3. Answer *any five* questions :

(a) Find the optimal solution of the following LPP by solving its dual :

Minimize $Z = 3x_1 + 2x_2$
subject to $7x_1 + 2x_2 \geq 30,\ 5x_1 + 4x_2 \geq 20,\ 2x_1 + 8x_2 \geq 16,\ x_1, x_2 \geq 0.$

6

(b) Solve the following transportation problem by Vogel's approximation method : **6**

	D_1	D_2	D_3	D_4	Available
O_1	15	20	13	21	30
O_2	22	15	19	14	12
O_3	16	12	14	31	18
O_4	24	23	15	30	10
Demand	25	20	15	10	

(c) Solve the following travelling salesman problem : **6**

	A	B	C	D	E
A	∞	7	6	8	4
B	7	∞	8	5	6
C	6	8	∞	9	7
D	8	5	9	∞	8
E	4	6	7	8	∞

(d) (i) State the maximin or minimax theorem.

(ii) Find the value of the following 2×2 game algebraically by using mixed strategies :

Player A \ Player B	B_1	B_2
A_1	2	3
A_2	4	-1

2+4

(e) Solve the game problem using the dominance property : **6**

$$\text{Player A} \begin{bmatrix} 0 & 2 & 0 & 1 & 1 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 1 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix} \quad \text{(columns: Player B)}$$

(f) Solve the following 2×4 game graphically : **6**

Player A \ Player B	B_1	B_2	B_3	B_4
A_1	6	5	2	3
A_2	1	2	6	3

(g) Solve the following game problem by using linear programming method : **6**

	B_1	B_2	B_3
A_1	1	0	-2
A_2	0	3	-2

(h) If X be any feasible solution to a primal problem and V be any feasible solution to its dual problem, then prove that $CX \leq b'V$, where C and b are respectively the cost vector and the requirement vector of the primal problem. **6**

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