UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — HONOURS

Paper: DSCC-9

(Probability and Statistics)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Probability]
(Marks : 45)

1. Answer *any five* questions : 3×5

(a) Consider events A and B such that $P(A) = \frac{1}{4}$, $P(B \mid A) = \frac{1}{2}$, $P(A \mid B) = \frac{1}{4}$. Find $P(\bar{A} \mid \bar{B})$.

(b) Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let X be the event that A is an invertible matrix, then find the probability $P(X)$.

(c) The probability of hitting a target is 0.0005. How many times one has to fire so that the probability of hitting a target at least once is more than 80%?

(d) Calculate the variance of normal distribution.

(e) Consider the distribution function of X given by $F(X) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - \frac{1}{4}e^{-x} & \text{for } x \geq 0 \end{cases}$. Determine $P(X > 0)$.

(f) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant K $(0 < K < 1)$ is $K(1 - \log_e K)$.

(g) Find the characteristic function of a Poisson μ-variate.

(h) Show that X and $Y - \rho\frac{\sigma_y}{\sigma_x}X$ are uncorrelated random variables, where the symbols have their usual meaning.

Answer *any five* questions :

2. (a) From an urn containing 3 white and 5 black balls, 4 balls are transferred to an empty urn. From the second urn 2 balls are drawn and they happen to be white. What is the probability that the third ball drawn from the same urn will be white? **6**

(b) Find the probability of obtaining a sum of 14, with results of throwing 3 dice. **6**

(c) Define distribution function of a random variable and show that it is a monotonic increasing function. If X is a random variable and a is a real number such that $P(X = a) > 0$, show that the distribution function of X is discontinuous at the point a. **2+2+2**

(d) If X is a $N(0, 1)$ variate, then find the distribution of

(i) $Y = \frac{1}{2}X^2$

(ii) $Y = e^X$. **4+2**

(e) In the equation $x^2 + 2x - q = 0$, q is a random variable uniformly distributed over the interval $(0, 2)$. Find the distribution function of the larger root. **6**

(f) The density function of a two-dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} Kxy(x+y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$. Find

(i) the value of K.

(ii) $P\left(\frac{1}{3} < Y \le \frac{2}{3} \middle| \frac{1}{2} < X \le \frac{3}{4}\right)$. **2+4**

(g) How many independent Bernoulli trials are required to ensure with a probability not less than 0.975, the validity of the inequality $\left|\frac{1}{n}X_n - p\right| < 0.1$, where X_n is the number of successes in n trials, where $p = \frac{1}{2}$ is the probability of a success in one trial? **6**

(h) A continuous random variable X is distributed over the interval $(0, 1)$ with pdf $ax^2 + bx$, where a and b are two constants. If $E(X) = 0.5$, then find the values of a and b. Hence find the third central moment of X. **3+3**

Group - B

[Statistics]

(Marks : 30)

3. Answer *any three* questions : 3×3

(a) Define an estimator. When is an estimator called unbiased and consistent?

(b) Find a 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample $(2.3, -0.2, -0.4, -0.9)$. $\left[\text{Given : } \frac{1}{\sqrt{2\pi}}\int_{1.960}^{\infty} e^{-t^2/2}dt = 0.025\right]$.

(c) Given the probability density function $f(x,\theta) = \frac{1}{\theta}, 0 \le x \le \theta$. Find the probability of Type-II error to test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ in the critical region $1 \le x \le 1.5$. Also find the power of the test.

(d) A company sells ice cream as packs of 500 gm. A sample of 16 packs is taken for inspection and sample mean is found to be 480 gm with a standard deviation of 30 gm. Does the sample differ significantly from the intended weight of 500 gm? [Given $t_{0.025,15} = 2.13$].

(e) If the density function of a population is given by $\frac{x^{l-1}e^{-\frac{x}{\alpha}}}{\alpha^l \Gamma(l)}$, $0 < x < \infty$, l being a known constant; estimate the parameter α by the method of maximum likelihood.

Answer *any three* questions :

4. (a) What is meant by the confidence interval for a parameter of a distribution? Find the confidence interval for the variance of a normal (m, σ) population on the basis of a random sample of size n and a confidence coefficient $1 - \alpha$. **2+5**

(b) Nine patients to whom a certain drug is administered, registered the following rise in blood pressure in mm of Hg : 3, 7, 4, −1, −3, 6, −4, 1, 5. Test the hypothesis that the drug did not raise blood pressure at 10% significance level, assuming that the sample is chosen from a normal population. Given $P(t > 1.86) = 0.05$ for eight degrees of freedom. **7**

(c) A die is thrown 150 times with the following data :

No. turned up	1	2	3	4	5	6
Frequency	19	23	28	17	32	31

Are the data consistent with the hypothesis that the die is unbiased? [Given $\chi^2_{0.05,5} = 11.07$] **7**

(d) (i) A bivariate sample of size 11 gave the result $\bar{x} = 7$, $S_x = 2$, $\bar{y} = 9$, $S_y = 4$ and $r = 0.5$. It was found that one of the sample value $(x = 7, y = 9)$ was inaccurate and rejected. How that value of r will be affected by the rejection?

(ii) Show that correlation coefficient is independent of change of origin. **5+2**

(e) Fit a second degree parabola $y = a + bx^2$, to the following data by the method of least square.

x	0	1	2	3	4
y	1	5	10	22	38

Estimate y when $x = 2.5$. **6+1**

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