UNIVERSITY OF CALCUTTA · Previous Year Question Paper

# 2025

## MATHEMATICS — MDC

**Paper: CC-1**

**(Calculus, Geometry and Vector Analysis)**

**Full Marks: 75**

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise:
https://palmaths.com/cu-pyqs/2025/maths-mdc-sem-1-cc-1/

*Write the answers to each Group in a separate answer-book.*

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*Symbols have their usual meanings.*

### Group - A
(Calculus)
(Marks : 20)

**1. Answer *any four* questions :** $2 \times 4$

(a) Find $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.

(b) $y = 2\cos x(\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$.

(c) Evaluate $\int_0^{\pi/2} \sin^9 x \, dx$ using the reduction formula

$$I_n = \frac{n-1}{n} I_{n-2}, \text{ where } I_n = \int_0^{\pi/2} \sin^n x \, dx.$$

(d) Find $\dfrac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$.

(e) Find the range of $x$ in which the function $2x^3 + 3x^2 - 36x + 2026$ is monotonically increasing.

(f) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point where $x = 2$.

(g) Find the area bounded by curve $y = x^2$ and the line $y = 2x$.

**Answer *any three* questions :** $4 \times 3$

**2.** (a) If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$, $|x| < 1$, show that $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$.

(b) Find $a$ and $b$ such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$

(c) Show that $x > \log(1 + x) > x - \dfrac{x^2}{2}$, for $x > 0$.

(d) If $I_n = \int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx$, show that $2(n - 1)I_n = 1 + (n - 2)I_{n-1}$.

(e) Find the perimeter of $\left(\dfrac{x}{a}\right)^{\frac{2}{3}} + \left(\dfrac{y}{b}\right)^{\frac{2}{3}} = 1$.

(f) Find the volume of the solid obtained by revolving the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 1$, $x = 4$ about $x$-axis.

## Group - B
(Geometry)
(Marks : 35)

**3. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) Find the angle of rotation of the axes about the origin which transforms the equation $x^2 - y^2 = 4$ to $x'y' = 2$.

(b) Find the point on the conic $\frac{14}{r} = 3 - 8\cos\theta$ whose vectorial angle is $\frac{2\pi}{3}$.

(c) Find the equation of the sphere passing through origin and the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.

(d) Show that the equation $2x^2 + 3y^2 - 8x + 6y - 12z + 11 = 0$ represents an elliptic paraboloid.

**Answer *any five* questions :** $6 \times 5$

**4.** (a) If the normal to the hyperbola $xy = c^2$ at the point $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve again at the point $\left(ct_2, \frac{c}{t_2}\right)$, then show that $t_1^3 t_2 + 1 = 0$.

(b) Show that the straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(A - e)^2 + B^2 = 1$.

(c) If $PSQ$ and $PS'R$ be two focal chords of an ellipse through the foci $S$ and $S'$, then show that $\frac{SP}{SQ} + \frac{S'P}{S'Q}$ is independent of the position of $P$.

(d) Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to its canonical form and determine the nature of the conic.

(e) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point $(2, -4, 6)$.

(f) Find the equation of the cone whose vertex is origin and which contains the curve $x^2 - y^2 + 4x = 0$, $x + y + z = 3$.

(g) Find the equation of the cylinder generated by straight lines parallel to $\frac{x}{1} = \frac{y}{5} = \frac{z}{-2}$, the guiding curve being the conic $x = 0$, $y^2 = 6z$.

(h) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$.

(i) Find the nature of the surface represented by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$.

## Group - C
(Vector Analysis)
(Marks : 20)

**5. Answer *any four* questions :** $2 \times 4$

(a) For any two vectors $\vec{\alpha}$ and $\vec{\beta}$ show that $\left|\vec{\alpha}\times\vec{\beta}\right|^2+\left|\vec{\alpha}\cdot\vec{\beta}\right|^2=|\vec{\alpha}|^2|\vec{\beta}|^2$.

(b) If a particle is acted on by constant forces $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+5\hat{k}$ and is displaced from the point $(\hat{i}+2\hat{j}-3\hat{k})$ to $2\hat{i}-3\hat{j}+7\hat{k}$, then find the total work done by the forces.

(c) Three forces $2\hat{i}+3\hat{j}-5\hat{k}$, $-5\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-2\hat{j}+4\hat{k}$ act on a particle, find the resultant of the forces.

(d) Find the vector equation of a straight line passing through the points $(2,3,4)$ and $(-1,3,-2)$.

(e) Find the volume of the tetrahedron whose vertices are $(1,1,0)$, $(1,0,1)$, $(0,1,1)$ and $(1,1,1)$.

(f) If $\vec{r}=(3t,3t^2,2t^3)$, find $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right]$.

(g) If $\vec{F}(t)=3t^2\hat{i}+t\hat{j}-t^3\hat{k}$, then find $\int_1^2 \vec{F}(t)\times\frac{d^2\vec{F}(t)}{dt^2}\,dt$.

**Answer *any three* questions :**

**6.** (a) $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are three non-coplanar vectors, then show that $(\vec{\alpha}-\vec{\beta})\cdot(\vec{\beta}-\vec{\gamma})\times(\vec{\gamma}-\vec{\alpha})=-[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]$.
*[PALMATHS note: As printed; this scalar triple product is actually 0 for any three vectors.]* **4**

(b) Find the shortest distance between the straight lines $\vec{r}=\vec{\alpha}+t\vec{\beta}$ and $\vec{r}=\vec{\gamma}+s\vec{\delta}$, where $\vec{\alpha}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{\beta}=2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{\gamma}=\lambda\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{\delta}=3\hat{i}+4\hat{j}+5\hat{k}$. **4**

(c) If $\vec{u}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k}$ and $\vec{v}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$, then show that

(i) $\frac{d}{dt}(\vec{u}\cdot\vec{v})=-6$ at $t=1$

(ii) $\frac{d}{dt}(\vec{u}\times\vec{v})=7\hat{j}+3\hat{k}$ at $t=1$. **2+2**

(d) If $\frac{d\vec{a}}{dt}=\vec{\omega}\times\vec{a}$ and $\frac{d\vec{b}}{dt}=\vec{\omega}\times\vec{b}$, show that $\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)=\vec{\omega}\times\left(\vec{a}\times\vec{b}\right)$. **4**

(e) If $\vec{F}=xy\hat{i}-2xy^2\hat{j}+zxy^3\hat{k}$ and $\vec{G}=2x\hat{i}+y\hat{j}-zx^2\hat{k}$, then find $\frac{\partial^2}{\partial x\partial y}\left(\vec{F}\times\vec{G}\right)$ at $(1,1,-1)$. **4**

(f) If $\vec{r}(t)=(t-t^2)\hat{i}+2t^3\hat{j}-3\hat{k}$, find $\int_1^2 \vec{r}\,dt$. **4**

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