UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MDC

Paper: CC-2

(Basic Algebra)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
$\mathbb{Z}$-set of integers, $\mathbb{R}$-set of real numbers.

Group - A
(Marks : 25)

1. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the principal value of i^i.

(b) For three positive numbers a, b, c prove that $(b+c)(c+a)(a+b) \geq 8abc$.

(c) Find the condition so that the equation $x^3 + 3ax + b = 0$ has a factor of the form $(x-\alpha)^2$.

(d) Show that the equation $x^7 - 2x^4 + 3x^2 - 1 = 0$ has at least four imaginary roots.

Answer *any four* questions : 5×4

2. (a) If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$.

(b) If sum of two roots of a cubic equation $x^3 + px^2 + qx + r = 0$, is equal to the third one, prove that $p^3 + 8r = 4pq$.

(c) Show that $\cos\left(i \log \frac{a-ib}{a+ib}\right) = \frac{a^2-b^2}{a^2+b^2}$.

(d) If $x+y+z = 1$, prove that the least value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is 9 and $(1-x)(1-y)(1-z) > 8xyz$.

(e) If $u + iv = \cot(x+iy)$, then show that $u^2 + v^2 - 2u \cot 2x = 1$.

(f) Find the value of k for which the equation $x^4 + 4x^3 - 2x^2 - 12x + k = 0$ has four real and unequal roots.

(g) Solve by Cardan's method : $x^3 - 3x + 1 = 0$.

Group - B
(Marks : 25)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Two mappings $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x^3$ and $g(x) = 5x + 1$ for all $x \in \mathbb{R}$. Find the mapping $f \circ g$.

(b) Find the remainder when 7^{20} is divided by 4.

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(c) Find whether the relation $\mathbb{R} = \{(a,b) : ab \geq 0\}$ on the set of all integers $\mathbb{Z}$ is reflexive, symmetric and transitive.

(d) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$, where a, b, c, d are integers and m is positive integer.

Answer *any four* questions :

4. (a) A relation ρ on the set of integer $\mathbb{Z}$ is defined by $\rho = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : |a-b| \leq 7\}$. Is the relation reflexive, symmetric and transitive? Justify your answer. **5**

(b) Let $f : A \to B$ and $g : B \to C$ be two mapping. If $g \circ f : A \to C$ is bijective, prove that f is injective and g is surjective. **5**

(c) Prove that the mapping $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{5x+3}{2}$ is a bijective mapping. Find f^{-1}. **5**

(d) State Division algorithm. Prove that the square of an odd integer is of the form $(8k+1)$. **5**

(e) Using the principle of mathematical induction, prove that $\left(10^{2n-1}+1\right)$ is divisible by 11, for all $n \in \mathbb{N}$. **5**

(f) Find all solution of the system of the congruence :

$$x \equiv 2 \pmod{3}, \quad x \equiv 2 \pmod{5}, \quad x \equiv 3 \pmod{7}.$$

5

(g) (i) Using Euclidean algorithm express $\gcd(24, 36)$ as $24u + 36v$, where u, v are integers.

(ii) Prove that if p be a prime number and $p \mid ab$, then $p \mid a$ or $p \mid b$. **2+3**

Group - C
(Marks : 25)

5. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the value of k so that the vectors $(1,3,1)$, $(2,k,0)$ and $(0,4,1)$ are linearly dependent in $\mathbb{R}^3$.

(b) Express $(4,3,5)$ as a linear combination of $(0,1,3)$ and $(2,1,1)$.

(c) Check whether the following system is consistent or not : $x+y+z=9$, $3x-2y+4z=3$.

(d) Find the rank of the matrix $A = \begin{bmatrix} 2 & 2 \\ -3 & -3 \end{bmatrix}$ using definition.

Answer *any four* questions : 5×4

6. (a) Reduce the following matrix to Row-Echelon matrix and hence find its rank :

$$\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$$

(b) Find values of a and b for which the equations $x+y+z=1$, $2x+3y+5z=b$, $4x+5y+az=b^2$ have (i) unique solution, (ii) more than one solution, and (iii) no solution.

(c) Find all real values of λ for which the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & \lambda \\ 5 & 7 & 1 & \lambda^2 \end{pmatrix}$ is 2.

(d) Transform the system of equation $x + y + z = 1$, $2x + y + 4z = 1$ and $4x + y + 10z = 1$ to a matrix equation $AX = B$. Hence, solve the system.

(e) Show that the four vectors $\alpha = (1, 0, 0)$, $\beta = (0, 1, 0)$, $\gamma = (0, 0, 1)$ and $\delta = (1, 1, 1)$ are linearly dependent but any three of them are linearly independent.

(f) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then show that $A^2 - 4A - 5I = 0$.

(g) Find a spanning set of $\mathbb{R}^3$ containing the vectors $(1, 2, 1)$ and $(2, 1, 1)$.

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