UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MDC

Paper: CC-3

(Ordinary Differential Equations and Group Theory)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Ordinary Differential Equations]
(Marks : 45)

Answer *any nine* questions.

1. (a) Find the order and degree of the differential equation :

$$\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y.$$

(b) Obtain the differential equation of all straight lines passing through the origin. **2+3**

2. Solve : $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$. **5**

3. Reduce the equation : $xp^2 - 2yp + x + 2y = 0$, $\left(p = \frac{dy}{dx}\right)$ to Clairaut's form by the substitution $x^2 = u, y - x = v$ and hence solve it. Also find the singular solution (if it exists). **4+1**

4. Solve : $\frac{dy}{dx} - \frac{\tan y}{(1+x)} = (1+x)e^x \sec y$. **5**

5. Find the general and singular solution of $y = px + \frac{a}{p}$, where $\left(p = \frac{dy}{dx}\right)$. **3+2**

6. Solve : $\frac{dy}{dx} = \frac{x+2y}{2x+y}$. **5**

7. Discuss the loci of the differential equation : $xp^2 = (x-a)^2$. **5**

8. Solve by the method of undetermined coefficients : $(D^2+1)y = 2\cos x$, $D \equiv \left(\frac{d}{dx}\right)$. **5**

9. Solve by the method of variation of parameters : $\frac{d^2y}{dx^2} + y = \tan x$. **5**

10. Solve : $\frac{d^2y}{dx^2} + a^2y = \sec(ax)$, $(a \neq 0)$. **5**

11. Solve : $(D^2 - 9)y = e^{3x}\cos x$, where $D \equiv \frac{d}{dx}$. **5**

12. Solve the simultaneous system : $\frac{dx}{dt} = x - y$, $\frac{dy}{dt} = x + y$. **5**

13. Solve : $(x^2D^2 + xD)y = x^2$. **5**

14. Show that, the equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$, $(0 < x < 1)$, is exact and hence solve it. **5**

15. Solve : $\frac{d^2y}{dx^2} = x^2 \sin x$. **5**

16. Solve : $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{(xyz - 2x^2)}$. **5**

Group - B

[Group Theory]
(Marks : 30)

Answer *any six* questions.

17. Let $(G, \circ)$ be a group. Then show that,

(a) $(a^{-1})^{-1} = a, \forall a \in G$

(b) $(a \circ b)^{-1} = b^{-1} \circ a^{-1}, \forall a, b \in G$. **2+3**

18. Let $(G, \circ)$ be a group and H be a non-empty finite subset of G. Then prove that the necessary and sufficient condition that $(H, \circ)$ is subgroup of $(G, \circ)$ is $a \in H, b \in H \Rightarrow ab \in H$. **5**

19. Prove that, the centre $Z(G)$ of a group G is a normal subgroup of G. **5**

20. If $\alpha = (1\,2\,3)(4\,5)$ and $\beta = (1\,4)(2\,5)$, then find $\beta^{-1}\alpha\beta$. **5**

21. Prove that, any subgroup of a cyclic group is cyclic. **5**

22. Let G be an abelian group. Prove that, the subset $H = \{g \in G : g^2 = e\}$ forms a subgroup of G. **5**

23. State and prove the Lagrange's theorem. **5**

24. Let G be the set of real numbers except the integer 1. Let the operation $*$ in G be defined by $a * b = a + b - ab$ for $\forall a, b \in G$. Verify : $(G, *)$ is an Abelian group. **5**

25. Let $(G, \circ)$ be a group and $a, b \in G$. If $o(a) = 3$ and $a \circ b \circ a^{-1} = b^2$, find $o(b)$ if $b \neq e$. **5**

26. Show that A_3, the set of even permutations of $\{1, 2, 3\}$ is a cyclic group with respect to product of permutations. Find a generator of the cyclic group A_3. **3+2**

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