UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MDC

Paper: MN-1

(Calculus, Geometry and Vector Analysis (Maths as Minor in MDC))

Full Marks: 75

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise:
https://palmaths.com/cu-pyqs/2025/maths-mdc-sem-3-mn-1/

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) If $\lim_{x\to 0} \dfrac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of a.

(b) Evaluate $\int_0^{\pi/4} \tan^5 x\, dx$.

(c) If $y = (x^2-1)^n$ then show that, $(x^2-1)y_2 - 2(n-1)xy_1 - 2ny = 0$.

(d) Find $\dfrac{dy}{dx}$, when $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$.

(e) Find the value of K in order that $f(x) = \sin x - Kx$ is a decreasing function.

(f) Find the length of the arc of the parabola $y^2 = 4x$ between the points where $x = 1$.

(g) Find the area of the region bounded by $x = \pm 1$, $y = 0$ and $y = x^2$.

Answer *any three* questions : 4×3

2. (a) Find the value of a and b in order that $\lim_{x\to 0} \dfrac{ae^x + be^{-x} + 2\sin x}{\sin x + x\cos x} = 2$.

(b) If $x^y = y^x$, then show that $\dfrac{dy}{dx} = \dfrac{y(x\log_e y - y)}{x(y\log_e x - x)}$.

(c) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2y_2 + xy_1 + y = 0$. Hence prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

(d) Find the Reduction Formula for $\int_0^{\pi/2} \sin^n x\, dx$, then find the value of $\int_0^{\pi/2} \sin^7 x\, dx$.

(e) Show that the arc length of the curve $y = x^{3/2}$ from $(0,0)$ to $(4,8)$ is $\dfrac{8}{27}\left(10^{3/2} - 1\right)$ unit.

(f) Prove that the area bounded by $y^2 = 4x$ and its latus rectum is $\dfrac{8}{3}$ sq. unit.

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Transform the equation $x^2 + y^2 + 10x - 10y + 21 = 0$ on transforming the origin to $(-2, 2)$.

(b) Find the radius of the sphere : $3x^2 + 3y^2 + 3z^2 + 2x - 4y - 2z - 1 = 0$.

(c) Find the length of latus rectum of the conic $\frac{16}{r} = 4 - 5\cos\theta$.

(d) Show that the equation $9x^2 + 4xy + 6y^2 - 10x + 20y + 5 = 0$ represents an ellipse.

Answer *any five* questions : 6×5

4. (a) Reduce the equation $3x^2 - 8xy - 3y^2 + 10x - 6y + 8 = 0$ to its canonical form and determine the nature of the conic.

(b) Show that the straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$, if $(A - e)^2 + B^2 = 1$.

(c) A plane passes through a fixed point (α, β, γ) and cuts the coordinates axes in A, B, C. Prove that the locus of the centre of the sphere OABC is given by $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 2$.

(d) Find the equation of cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.

(e) Find the equation of the tangents to the circle $x^2 + y^2 + 8x + 10y - 4 = 0$ which are parallel to the straight line $x + 2y + 3 = 0$.

(f) Show that the locus of the point of intersection of a pair of perpendicular tangents to an ellipse is a circle.

(g) Find the equation of the sphere whose center lies on the line $\frac{x+1}{3} = \frac{y-3}{-4} = \frac{z+2}{5}$ and which passes through the point $(3, 4, 5)$ and $(-3, 0, 1)$.

(h) Find the equation of the generators of the hyperbolic paraboloid $16x^2 - 9y^2 = 4z$ passing through the point $(1, 0, 4)$.

(i) Show that $3x^2 + 5y^2 + 3z^2 + 2yz + 2zx + 2xy - 4x - 8z + 5 = 0$ represents a Central Conicoid. Find its centre.

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Show that the three vectors $4\hat{i} + 2\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $8\hat{i} + 7\hat{k}$ are coplanar.

(b) Prove that $\left[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$.

(c) Find the vector equation of the straight line passing through the point $\hat{i} + \hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - \hat{k}$.

(d) A force $\vec{F} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of force $\vec{F}$ about the point $(1, -2, 3)$.

(e) Show that $[\hat{i} + \hat{j} \;\; \hat{j} + \hat{k} \;\; \hat{k} + \hat{i}] = 2$.

(f) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t + 1)\hat{k}$ and $\vec{b} = 2t\hat{i} + \hat{j} - t\hat{k}$, then find the value of $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right)$ at $t = 0$.

(g) Find the work done in moving an object along a vector $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$. If the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$.

Answer *any three* questions : 4×3

6. (a) If $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are three non-coplanar vectors, then prove that $(\vec{\alpha} - \vec{\beta}) \cdot (\vec{\beta} - \vec{\gamma}) \times (\vec{\gamma} - \vec{\alpha}) = 0$.

(b) Find the vector equation of a plane passing through the points $(-2, 6, -6)$, $(-3, 10, -9)$ and $(-5, 0, -6)$.

(c) If $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when $t = 2$ and $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$, when $t = 3$, then show that $\int_2^3 \left(\vec{r} \cdot \frac{d\vec{r}}{dt}\right) dt = 10$.

(d) The position vector of a moving point of a space curve is given by $\vec{r} = \hat{i} - 4t^2\hat{j} + 3t^2\hat{k}$. Find the magnitude and direction of the velocity and acceleration at any time t.

(e) If a vector function $\vec{a}(t)$ possesses constant direction then show that $\vec{a} \times \frac{d\vec{a}}{dt} = \vec{0}$.

(f) If $\vec{\gamma} = (2a\cos t, 2a\sin t, bt^2)$ then prove that $[\dot{\vec{\gamma}}, \ddot{\vec{\gamma}}, \dddot{\vec{\gamma}}] = 8a^2bt$.

PALMATHS