UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MDC

Paper: CC-5

(Advanced Calculus)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
(Symbols have their usual meaning.)

Group - A
(Marks : 20)

1. Answer *any four* questions :

(a) Prove that a convergent sequence is bounded. Is the converse true? – Justify your answer. **3+2**

(b) Define a convergent sequence. Show that the sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$ converges to 2. **1+4**

(c) Prove that the sequence $\left(1+\frac{1}{n}\right)^n$ is a monotone increasing sequence and bounded above. **3+2**

(d) Show by applying Cauchy's convergence criterion, that the sequence $\{x_n\}$ defined by

$$x_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$$

does not converge. **5**

(e) Write the statement of Leibnitz's test of infinite series. Is the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ convergent? Justify your answer. **2+3**

(f) Test the convergence of the series $\sum \frac{n^2-1}{n^2+1}x^n$, $x > 0$. **5**

(g) Test the convergence of the infinite series $\frac{5}{2\cdot 2\cdot 4} + \frac{7}{4\cdot 3\cdot 5} + \frac{9}{6\cdot 4\cdot 6} + \ldots$. **5**

Group - B
(Marks : 25)

2. Answer *any five* questions :

(a) Let $f : \mathbb{R} \to \mathbb{R}$. If $\lim_{x\to c} f(x) = l$, then prove that $\lim_{x\to c} |f(x)| = |l|$. Is the converse true? Justify your answer. **2+3**

(b) (i) Using Cauchy's criterion prove that $\lim_{x\to 0} \cos\frac{1}{x}$ does not exist.

(ii) Given that $f(x) = |x|$, show that f is not differentiable at $x = 0$. **3+2**

(c) Evaluate :

(i) $\lim_{x\to 1}\dfrac{x^2-1}{x-1}$

(ii) $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$ $2\frac{1}{2}+2\frac{1}{2}$

(d) Let $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0. \end{cases}$ Check the continuity of f at $x=0$. **5**

(e) Let $f:[0,1]\to[0,1]$ be continuous on $[0,1]$. Prove that there exists a point $c\in(0,1)$ such that $f(c)=c$. **5**

(f) (i) State Rolle's theorem. Is Rolle's theorem applicable to the function $f(x)=\tan x$ in $[0,\pi]$? – Justify.

(ii) Let $f(x)=1+x$ if $x\leq 2$; $=5-x$ if $x>2$. Test the nature of the function $f(x)$ at $x=2$ as regards its differentiability. **(1+2)+2**

(g) (i) Show that the function $f(x)=|x-1|$ is continuous at $x=1$, but not differentiable at that point.

(ii) If $f(h)=f(0)+hf'(0)+\dfrac{h^2}{2!}f''(\theta h)$, $0<\theta<1$, find θ when $h=1$ and $f(x)=(1-x)^{5/2}$. **3+2**

(h) Prove that between any two real roots of $e^x\sin x=1$, there exists at least one real root of $e^x\cos x+1=0$. **5**

(i) (i) State the geometrical interpretation of Lagrange's Mean Value theorem.

(ii) Show that the maximum value of $x+\dfrac{1}{x}$ is less than its minimum value. **2+3**

Group - C
(Marks : 30)

3. Answer *any six* questions :

(a) Show that for $f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ double limit exists at $(0,0)$. What can you say about both repeated limits at $(0,0)$? **3+2**

(b) If $f(x,y)=\sqrt{|xy|}$, find $f_x(0,0)$ and $f_y(0,0)$. **5**

(c) State Schwarz's theorem. If $f(x,y)=xy$ when $|x|\geq|y|$; $=-xy$ when $|x|<|y|$, show that $f_{yx}(0,0)\neq f_{xy}(0,0)$. Which condition of Schwarz's theorem are not satisfied by f? **1+4**

(d) If $u=\tan^{-1}\dfrac{x^3+y^3}{x-y}$, apply Euler's theorem to prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\sin 2u$. Hence deduce that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u$. $\left[\text{assuming } \dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{\partial^2 u}{\partial y\partial x}\right]$. **2+3**

(e) If $x^2y^2+x^2+y^2-1=0$ determines y uniquely as a function of x near the point $(0,1)$, calculate $\dfrac{dy}{dx}$ at that point. **5**

(f) If $z=e^{xy^2}$, $x=t\cos t$, $y=t\sin t$, compute $\dfrac{dz}{dt}$ at $t=\dfrac{\pi}{2}$. **5**

(g) Find $df(1,2)$ if $f(x,y)=x^2+xy+y^2-4\log_e x-10\log_e y$. **5**

(h) If $f(x,y)$ is continuous at (a,b), then prove that the function $f(x,b)$ is continuous at $x=a$. If $f(x,b)$ and $f(a,y)$ are continuous at $x=a$ and $y=b$ respectively, does it imply continuity of $f(x,y)$ at (a,b)? Justify your answer. **2+3**

(i) (i) Use ϵ–δ definition of limit to prove that $\lim\limits_{(x,y)\to(1,-1)} \dfrac{x^2-y^2}{x+y} = 2$.

(ii) If $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2} & \text{when } x^2+y^2 \neq 0 \\ 0 & \text{when } x^2+y^2 = 0 \end{cases}$, show that $f_x(0,0) = 1$ and $f_y(0,0) = -1$.

2+3

(j) If $v = f(x,y)$ and $x = e^u \cos t$, $y = e^u \sin t$, then show that

$$\left(\frac{\partial v}{\partial u}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 = (x^2+y^2)\left\{\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\}.$$

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