UNIVERSITY OF CALCUTTA · Previous Year Question Paper

# 2025

## MATHEMATICS — MDC

**Paper: MN-2**

**(Basic Algebra (Maths as Minor in MDC))**

**Full Marks: 75**

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*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

### Group - A
(Marks : 25)

**1. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) If $z = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$, then find $\frac{1}{z}$.

(b) Form the equation whose roots are $1, -\frac{1}{2}, 5$.

(c) Show that $(b+c)(c+a)(a+b) > 8abc$, if $a, b, c$ are positive and not all equal.

(d) Find the condition that the $x^3 - px^2 + qx - r = 0$ should have roots in geometric progression.

**Answer *any four* questions :** $5 \times 4$

**2.** (a) If $x$ is real, prove that $i\log\frac{1+ix}{1-ix} = -2\tan^{-1}x$.

(b) Find the equation whose roots are the roots of the equation $x^3 + 3x^2 - 8x + 1 = 0$, each diminished by 4.

(c) Solve by Cardan's method, $x^3 - 12x + 65 = 0$.

(d) Solve the equation, $x^3 - 3x^2 + 4 = 0$, two roots being equal.

(e) Prove that the minimum value of $x^2 + y^2 + z^2$ is 27, where $x, y, z$ are positive real variables satisfying the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

(f) Find the general solution of $\sin z = 2$, where $z$ is a complex number.

(g) If $\alpha, \beta, \gamma$ be the roots of the equation, $x^3 - px^2 + qx - r = 0$, then find the equation whose roots are $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$.

### Group - B
(Marks : 25)

**3. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) Prove that $19^{20} \equiv 1 \pmod{181}$.

(b) A relation $R = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : 2a + 3b \text{ is divisible by } 5\}$ is defined on $\mathbb{Z}$, the set of all integers. Examine if it is an equivalence relation on $\mathbb{Z}$.

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(c) Use the principle of mathematical induction to prove that $1+3+5+\cdots+(2n-1)=n^2$ for all $n \in \mathbb{N}$, $\mathbb{N}$ is the set of all natural numbers.

(d) Find two integers $u$ and $v$ satisfying $63u+55v=1$.

**Answer *any four* questions :**

**4.** (a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$, $x \in \mathbb{R}$ is surjective but not injective. **5**

(b) Prove that, product of any three consecutive integers is divisible by 6. **5**

(c) Show that the function, $f : \mathbb{Q} \to \mathbb{Q}$, where $f(x) = 5x+2$ $(x \in \mathbb{Q})$ is both one-one and onto. **3+2**

(d) Prove that $5^{2n}-1$ is divisible by the 24 by the principle of mathematical induction. **5**

(e) Find the solution of each of the following equations and also the simultaneous equation, $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 7$. **5**

(f) Examine if the following relations $S$ on the set $\mathbb{Z}$ of all integers are equivalence relation : $S = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : |a-b| \leq 3\}$ **5**

(g) Prove that for $n > 3$, the integers $n$, $n+2$, $n+4$ cannot be all primes. **5**

## Group - C
(Marks : 25)

**5. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) If $\alpha = (1,1,2)$, $\beta = (0,2,1)$, $\gamma = (2,2,4)$, determine whether $\alpha$ is a linear combination of $\beta$ and $\gamma$.

(b) Determine $k$ so that the set $\{(1,2,1),(k,3,1),(2,k,0)\}$ becomes linearly independent in $\mathbb{R}^3$.

(c) Find a $2 \times 2$ matrix $A$, such that $A\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix} = \begin{bmatrix}2 & 3\\ 9 & 1\end{bmatrix}$.

(d) Find the rank of the matrix $A = \begin{bmatrix}1 & 3 & 2\\ 7 & 21 & 14\\ 4 & 12 & 8\end{bmatrix}$.

**Answer *any four* questions :** $5 \times 4$

**6.** (a) Determine the condition for which the system, $x+y+z=1, x+2y-z=b, 5x+7y+az=b^2$ has (i) unique solution, (ii) no solution, (iii) infinite many solutions.

(b) Reduce the following matrix to row echelon form, and find the rank of the matrix $\begin{pmatrix}1 & 0 & -4\\ 0 & 0 & 2\\ 0 & 1 & -2\end{pmatrix}$.

(c) Show that the equations,

$$x+2y-z=3, \quad 3x-y+2z=1, \quad 2x-2y+3z=2, \quad x-y+z=-1$$

are consistent.

(d) Show that the subset $S = \{(1,0,0),(0,1,0),(0,0,1)\}$ of $\mathbb{R}^3$ spans $\mathbb{R}^3$.

(e) Transform the system of linear equations $2x-3y+z=1$, $x+2y-3z=4$, $4x-y-2z=8$ to a matrix equation, $AX=B$ and hence solve the system using the properties of matrix.

(f) If $A$ and $B$ are square matrices of the same order such that $A$, $B$ and $(A+B)$ are invertible. Prove that $A(A+B)^{-1}B = B(A+B)^{-1}A$, where $A^{-1}$ is inverse matrix $A$.

(g) Write the matrix $E = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$.

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