UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MINOR

Paper: MN-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
[Calculus]
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Find the value of $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.

(b) Find $\dfrac{dy}{dx}$ when $y = x^{\sinh x}$.

(c) Find the values of k for which the function $f(x) = kx - \sin x$ increases.

(d) If $y = Ae^{kx} + Be^{-kx}$, then show that $y_2 + k^2 y = 0$, where A, B and k are constants.

(e) Evaluate $\int_0^{\pi/4} \tan^5 x \, dx$, using the reduction formula : $\int_0^{\pi/4} \tan^n x \, dx = \dfrac{1}{n-1} - I_{n-2}$.

(f) Find by the method of double integral, the area of a circle of radius r.

(g) Show that the length of the circumference of the circle $x^2 + y^2 = 9$ is 6π (using calculus).

Answer *any three* questions : 4×3

2. (a) Find the values of a and b in order that $\lim_{x \to 0} \dfrac{x(1 + a\cos x) - b \sin x}{x^3} = 1$.

(b) If $y = \tan^{-1} x$, then show that $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

(c) Write down the Reduction formula for $\int \sin^n x \, dx$, hence evaluate $\int \sin^4 x \, dx$.

(d) Find the length of perimeter of the curve given by $x = \dfrac{1 - t^2}{1 + t^2}$, $y = \dfrac{2t}{1 + t^2}$.

(e) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(f) Find the volume of the solid generated by revolution of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ about the X-axis.

Group - B
[Geometry]
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

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(a) Find the transformed equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$ referred to parallel axes through the point $(-1, 2)$.

(b) Determine the nature of the conic $\frac{5}{r} = 3 + 6\cos\theta$ and also find the length of its latus rectum.

(c) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y + 6z - 2 = 0$.

(d) Find the rectangular Cartesian coordinates of the point whose polar coordinate is $\left(3, \frac{\pi}{3}\right)$.

Answer *any five* questions : 6×5

4. (a) Reduce the equation $4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$ to its cannonical form and hence determine the nature of the conic.

(b) If the straight line $r\cos(\theta - \alpha) = p$ touches the parabola $\frac{l}{r} = 1 + \cos\theta$, show that $p = \frac{l\sec\alpha}{2}$.

(c) Find the equation of the locus of the points of intersection of mutually perpendicular tangents to the circle $x^2 + y^2 = r^2$.

(d) A sphere passes through the origin and meets the coordinate axes at A, B, C so that the volume of the tetrahedron OABC is K. Find the locus of the centre of the sphere.

(e) Find the equation of the right circular cone whose vertex is the origin, whose axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi-vertical angle of $30°$.

(f) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$ and the guiding curve is $4x^2 + y^2 = 1, z = 0$.

(g) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$ passing through the point $(2, 3, -4)$.

(h) Find the equation of the right circular cylinder whose radius is 10 units, axis passes through the point $(1, 2, 3)$ and is parallel to the straight line $\frac{x-5}{2} = \frac{y}{-1} = \frac{z-7}{2}$.

(i) Show that the equation $3x^2 - y^2 - z^2 + 6yz - 6x + 6y - 2z - 2 = 0$ is a central conicoid. Find its centre.

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer *any four* questions : 2×4

(a) Find the value of x for which the vectors $x\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$ are coplanar.

(b) Find the vector equation of the line passing through the point $\hat{i} + \hat{j} + \hat{k}$ and parallel to the vector $3\hat{i} + 2\hat{j} - \hat{k}$.

(c) If $|\vec{\alpha}| = 2$, $|\vec{\beta}| = 3$ and $\vec{\alpha} \cdot \vec{\beta} = 2$, find $\vec{\alpha} \times \vec{\beta}$.

(d) Find the angle between two vectors $2\hat{i} + \hat{j} - 3\hat{k}$ and $2\hat{i} + 3\hat{j} + \hat{k}$.

(e) If $\vec{a}$ and $\vec{b}$ be two vectors and let k be a given scalar, find all vectors $\vec{r}$ such that $\vec{a} \times \vec{r} = \vec{b}$ and $\vec{a} \cdot \vec{r} = k$.

(f) If $\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$, find $\left|\frac{d\vec{r}}{dt}\right|$.

(g) Prove that for any proper vector $\vec{\alpha}$, $\hat{i} \times (\vec{\alpha} \times \hat{i}) + \hat{j} \times (\vec{\alpha} \times \hat{j}) + \hat{k} \times (\vec{\alpha} \times \hat{k}) = 2\vec{\alpha}$.

Answer *any three* questions : 4×3

6. (a) Prove by vector method, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ in a triangle $\triangle ABC$.

(b) Find the vector equation of a plane through the three points $(2, -1, 1)$, $(3, 2, -1)$ and $(-1, 3, 2)$.

(c) A particle, acted on by constant forces $2\hat{i} + 3\hat{j} - \hat{k}$ and $3\hat{i} - 3\hat{j} + 5\hat{k}$ is displaced from the point $A(1, 3, 2)$ to the point $B(4, 5, -1)$. Find the work done by the force on the particle.

(d) Prove that $\left[\vec{\alpha} + \vec{\beta} \;\; \vec{\beta} + \vec{\gamma} \;\; \vec{\gamma} + \vec{\alpha}\right] = 2\left[\vec{\alpha} \; \vec{\beta} \; \vec{\gamma}\right]$.

(e) A particle move along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. Find the components of velocity and acceleration at time $t = 1$, in the direction $(\hat{i} - 3\hat{j} + 2\hat{k})$.

(f) If $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{C} = (\sin t)\hat{i} - (\cos t)\hat{j}$, then find $\frac{d}{dt}\left(\vec{A} \times \vec{C}\right)$.

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