UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MINOR

Paper: MN-2

(Basic Algebra)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Throughout the question paper $\mathbb{R}$ denotes the set of real numbers.
Other symbols have their usual meaning.

Group - A
(Marks : 25)

1. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Express $\dfrac{i^5 + i^9 + i^{14}}{i^6 + i^{11} + i^{13}}$ in $a + ib$ form. $\left(i = \sqrt{-1}\right)$

(b) For any two positive real numbers a and b, prove that $a + b + \dfrac{1}{a} + \dfrac{1}{b} \geq 4$.

(c) If α, β and γ are the roots of the equation $x^3 + x + 1 = 0$, then find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$.

(d) Find the principal value of $\arg z$, where $z = 1 + i \tan \dfrac{3\pi}{5}$.

Answer *any four* questions :

2. (a) If n is a positive natural number, then prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos \dfrac{n\pi}{4}$. **5**

(b) Find the principal value of i^i and also show that all the values of i^i form a G.P. with common ratio $e^{-2\pi}$. **2+3**

(c) If α, β and γ are the roots of the equation $x^3 - 6x - 4 = 0$, find the equation whose roots are $\alpha + 2$, $\beta + 2$ and $\gamma + 2$. Hence, show that -2 is a root of the original equation. **4+1**

(d) Solve the equation $4x^4 + 20x^3 + 35x^2 + 25x + 6 = 0$, given that the roots of the equation are in arithmetic progression. **5**

(e) Solve $x^3 - 3x^2 + 3 = 0$ by Cardan method. **5**

(f) If a, b are positive numbers and $a + b = 6$, then prove that $\left(a + \dfrac{1}{a}\right)^2 + \left(b + \dfrac{1}{b}\right)^2 \geq \dfrac{199}{9}$. **5**

(g) Prove that $(n!)^3 \geq n^n \left(\dfrac{n+1}{2}\right)^{2n}$, $\forall n \in \mathbb{N}$. **5**

Group - B
(Marks : 25)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

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(a) If $\rho = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : ab > 0\}$, then show that ρ is symmetric and transitive but not reflexive.

(b) $f : A \to B$ function defined as follows :

$$f(x) = \frac{x+1}{x-1},\ \forall x \in A \text{ where } A = \mathbb{R} \setminus \{1\} \text{ and } B = \mathbb{R} \setminus \{1\},$$

[$\mathbb{R}$ is the set of real numbers]. Check whether f^{-1} exists? Give proper reason.

(c) Find $\sigma(n)$, where $n = 1225$ and $\sigma(n)$ denotes the sum of all positive divisors of n.

(d) Prove that the number of primes is infinite.

Answer *any four* questions :

4. (a) Solve for x using Chinese remainder theorem :

$$x \equiv 3 \pmod 7, \quad x \equiv 1 \pmod{11}, \quad x \equiv 4 \pmod 5$$

5

(b) If $\mathbb{Z}^*$ be the set of non-negative integers and $f : \mathbb{Z} \to \mathbb{Z}^*$ be a mapping defined by $f(x) = \frac{1}{2}[x + |x|], \forall x \in \mathbb{Z}$; then check whether $f(x)$ is bijective or not? **5**

(c) (i) Find the digit in the unit place of 17^{17}.

(ii) For two integers a and b, if $7a \equiv 7b \pmod 5$, then show that $a \equiv b \pmod 5$. **3+2**

(d) Prove by mathematical induction : $1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n2^n = (n-1)2^{n+1} + 2, \forall n \in \mathbb{N}$. **5**

(e) Prove that $\phi(2n) = \phi(n)$, when n is odd and $\phi(2n) = 2\phi(n)$, when n is even. **5**

(f) Give examples of :

(i) an injective mapping from $\mathbb{N}$ to $\mathbb{Z}$.

(ii) a surjective mapping from $\mathbb{N}$ to $\mathbb{Z}$. **2+3**

(g) Using Euclidean Algorithm, find two integers u and v such that $128u + 66v = 2$. **5**

Group - C
(Marks : 25)

5. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Prove or disprove : the set of vectors $\{(1,2,3),(2,3,1),(3,1,2)\}$ in $\mathbb{R}^3$ linearly dependent.

(b) Determine the values of 'a' so that the rank of the matrix $\begin{pmatrix} 1 & a \\ a & 2 \end{pmatrix}$ can't exceed 1.

(c) $(1,2,3) \in \mathbb{R}^3$. Express $(1,2,3)$ as linear combination of the vectors $(1,2,1), (0,1,0), (0,0,1)$.

(d) Examine the consistency of the following system of linear equations :

$$x + y + 2z = 1, \quad 2x + y + 2z = 3, \quad 3x + 2y + 3z = 7$$

Answer *any four* questions :

6. (a) Find the value of K for which the following system of equations has (i) no solution (ii) more than one solution (iii) only one solution :

$$x + y - z = 1, \quad 2x + 3y + Kz = 3, \quad x + Ky + 3z = 2$$

5

(b) (i) If A and B are two square matrices such that $AB = A$ and $BA = B$, show that $A^2 = A$ and $B^2 = B$.

(ii) If $A = \begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix}$, then show that, for any 2×2 matrix B, $AB = BA$. **3+2**

(c) Find rank of the following matrix by reducing it to row-reduced Echelon form :

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

5

(d) Show that the vectors in $S = \{(1,2,0), (2,1,3), (1,1,1), (2,3,1)\}$ are linearly dependent set in $\mathbb{R}^3$. Find a subset S_1 of S s.t. $L(S_1) = L(S)$ and the vectors in S_1 are linearly independent. **5**

(e) Express the equations $2x - y = 3$, $3y - 2z = 5$ and $2z - x = -4$ in the $AX = B$ form and solve. **2+3**

(f) Find the parametric form of the solution set of :

$$x_1 - 8x_3 - 7x_4 = 0, \qquad x_2 + 4x_3 + 3x_4 = 0$$

5

(g) If $P = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$, find the rank of the matrix $P^3 + 2P$. **5**

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