UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MINOR

Paper: MN-1

(Calculus, Geometry and Vector Analysis)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A
(Calculus)
(Marks : 20)

1. Answer *any four* questions : 2×4

(a) Using L'Hôpital's Rule, evaluate $\lim_{x\to 0} \dfrac{\sin(x) - x}{x^3}$.

(b) Find the area bounded by the curve $x = y^2$ and the lines $x = 0$ and $x = 2$.

(c) Find the volume of revolution for $y = \sin(2x)$, $0 \le x \le \dfrac{\pi}{2}$, about x-axis.

(d) If $x^y = y^x$, then find $\dfrac{dy}{dx}$.

(e) If $y = \left(x + \sqrt{1+x^2}\right)^m$, then prove that $(1+x^2)y_2 + xy_1 - m^2 y = 0$.

(f) If $I_n = \displaystyle\int_0^{\pi/2} \sin^n\theta\, d\theta = \frac{n-1}{n} I_{n-2}$, then evaluate $\displaystyle\int_0^{\pi/2} \sin^4 x\, dx$.

(g) Find the perimeter of the curve $r = a(\cos\theta + \sin\theta)$, $0 \le \theta \le \pi$.

Answer *any three* questions : 4×3

2. (a) Show that the function $f(x) = |x|$ is not differentiable at $x = 0$.

(b) Find the arc length of the curve $x = a\cos(t)$, $y = a\sin(t)$; $0 \le t \le \dfrac{\pi}{2}$.

(c) Find the values of a and b such that $\lim_{x\to 0} \dfrac{x(1 - a\cos x) + b\sin x}{x^3} = \dfrac{1}{3}$.

(d) If $y = e^{m\sin^{-1}x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2+n^2)y_n = 0$.

(e) If $I_{m,n} = \displaystyle\int_0^{\pi/2} \sin^m x \cos^n x\, dx$, show that

$$I_{m,n} = \frac{n-1}{m+n} I_{m,n-2}.$$

Hence, evaluate $\displaystyle\int_0^{\pi/2} \sin^4 x \cos^8 x\, dx$.

(f) Find the surface area generated by revolving about y-axis the part of astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in the first quadrant.

Group - B
(Geometry)
(Marks : 35)

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Find the centre of the circle where the plane $x - 2y + 2z = 3$, intersects the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z = 45$.

(b) Show that $2x - 2y + 3z = 9$, touches $x^2 + 2y^2 + 3z^2 = 9$, at the point $(2, -1, 1)$.

(c) What will be the form of the equation $x^2 - y^2 + 3x + 2y = 1$, if the co-ordinate axes are rotated through an angle $\frac{\pi}{2}$?

(d) Find the point of intersection of two conics $\frac{1}{r} = 1 + \cos\theta$ and $\frac{3}{r} = 1 - \cos\theta$.

Answer *any five* questions : 6×5

4. (a) If the straight line $r\cos(\theta - \alpha) = p$ touches the parabola $\frac{l}{r} = 1 + \cos\theta$, show that $p = \frac{l}{2}\sec\alpha$.

(b) The normal at any point P on the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ meets the planes $x = 0$, $y = 0$, $z = 0$, respectively at G_1, G_2, G_3. Show that $PG_1 : PG_2 : PG_3 = a^2 : b^2 : c^2$.

(c) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$.

(d) Show that the two circles

$$x^2 + y^2 + z^2 - y + 2z = 0, \quad x - y + z - 2 = 0; \text{ and}$$
$$x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, \quad 2x - y + 4z - 1 = 0;$$

lie on the same sphere and find its (the sphere) equation.

(e) Show that the equation $7x^2 - 2xy + 7y^2 + 22x - 10y + 7 = 0$ represents an ellipse. Find it's centre.

(f) Show that the equation of the cone which passes through the co-ordinate axes and the straight lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{3} = \frac{y}{2} = \frac{z}{-1}$ is $3yz + 10zx + 6xy = 0$.

(g) Find the equations to the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$ which passes through the line $x + 9y - 3z = 0$ and $3x - 3y + 6z - 5 = 0$.

(h) Find the nature of the quadric surface given by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z = 5$.

(i) If r_1 and r_2 be two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2\cos^2\theta}$, then prove that $\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Group - C
(Vector Analysis)
(Marks : 20)

5. Answer *any four* questions : 2×4

(a) If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$, then show that $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

(b) If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three vectors such that $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$ and $|\vec{\alpha}| = 3$, $|\vec{\beta}| = 5$, $|\vec{\gamma}| = 7$, find the angle between $\vec{\alpha}$ and $\vec{\beta}$.

(c) Forces 6, 7, 2 pounds weight act on a particle along the vectors with co-ordinates $(6, 2, 3)$, $(3, -2, 6)$ and $(2, -3, -6)$ respectively. If the particle be displaced from the point $(2, -1, -3)$ to the point $(5, -1, 1)$, then show that the work done is $53\frac{4}{7}$ feet pounds weight, the unit of length being one foot.

(d) If $\vec{r} = t\hat{i} + t^2\hat{j} + \frac{2}{3}t^3\hat{k}$, then find $k = \dfrac{|\dot{\vec{r}} \times \ddot{\vec{r}}|}{|\dot{\vec{r}}|^3}$.

(e) Determine all possible values of λ for which the vector $\vec{\alpha} = \lambda(2\hat{i} - 2\hat{j} + 6\hat{k})$ is a unit vector.

(f) Show that the vector $\left(\vec{\beta} - \dfrac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}|^2}\vec{\alpha}\right)$ is perpendicular to the vector $\vec{\alpha}$.

(g) Find the vector equation of a plane through the point $(4, 3, -1)$ and perpendicular to the vector $(3\hat{i} - 4\hat{j} + \hat{k})$.

Answer *any three* questions : 4×3

6. (a) Find a unit vector in the plane of the vectors $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and is perpendicular to the vector $\vec{c} = 3\hat{i} + \hat{j} + 5\hat{k}$.

(b) If $\vec{r} = 2t\hat{i} + \frac{1}{3}t^3\hat{j} + \frac{1}{5}t^5\hat{k}$, then find the value of $\left(\dfrac{d\vec{r}}{dt} \times \dfrac{d^2\vec{r}}{dt^2}\right) \cdot \dfrac{d^3\vec{r}}{dt^3}$ at $t = 1$.

(c) Prove that $\left[\vec{\alpha} + \vec{\beta},\ \vec{\beta} + \vec{\gamma},\ \vec{\gamma} + \vec{\alpha}\right] = 2\left[\vec{\alpha}, \vec{\beta}, \vec{\gamma}\right]$.

(d) Find the angle between the tangents to the curve $x = t$, $y = t^2$, $z = t^3$ at the points $t = 1$ and $t = -1$.

(e) If a force $\vec{F} = 2\hat{i} + 4\hat{j} - \hat{k}$ moves a particle from the point A to B, where the position vectors of A and B are $\hat{i} + \hat{j} + 2\hat{k}$ and $3\hat{i} - \hat{j} - \hat{k}$ respectively, then find the work done by the force.

(f) Evaluate : $\displaystyle\int_2^3 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$, where $\vec{r} = t^3\hat{i} + 2t^2\hat{j} + 3t\hat{k}$.

[PALMATHS note: Q. 6(e) and 6(f) are typed from the Bengali version of the paper, as the last page of the English version was missing from our copy.]

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