UNIVERSITY OF CALCUTTA · Previous Year Question Paper

# 2025

## MATHEMATICS — MINOR

**Paper: MN-2**

**(Basic Algebra)**

**Full Marks: 75**

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*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*

### Group - A

**1. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) Find the cube roots of $-i$.

(b) If $\alpha, \beta, \gamma$ are the roots of the equation $2x^3 + x^2 - 2x - 1 = 0$, then find the value of $\sum \alpha^2$.

(c) Find the nature of roots of the equation $x^4 + x^2 + x - 1 = 0$ using Descarte's Rule of signs.

(d) If $a, b, c, d$ are positive real numbers then, prove that $(a+b+c+d)(a^3+b^3+c^3+d^3) \geq 16abcd$.

**Answer *any four* questions :** $5 \times 4$

**2.** (a) If $x$ is real, prove that $i \log \dfrac{1+ix}{1-ix} = -2\tan^{-1} x$.

(b) If $\alpha, \beta$ are the roots of the equation $t^2 + 2t + 4 = 0$ and $m$ is a positive integer, prove that $\alpha^m + \beta^m = 2^{m+1} \cos \dfrac{2m\pi}{3}$.

(c) Find the equation whose roots are the roots of the equation $x^3 + 3x^2 - 8x + 1 = 0$, each increased by 1.

(d) Solve : $x^3 - 6x - 4 = 0$ by Cardan's Method.

(e) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 - px - q = 0$, find the value of $(\alpha^2 + \beta^2) \times (\beta^2 + \gamma^2) \times (\gamma^2 + \alpha^2)$.

(f) Solve the equation by Ferrari's method $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

(g) If $a_1, a_2, \ldots, a_n$ are unequal positive real numbers in A.P., show that $\dfrac{1}{a_1} + \dfrac{1}{a_2} + \cdots + \dfrac{1}{a_n} > \dfrac{2n}{a_1 + a_n}$.

### Group - B

**3. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) $R$ is a relation on the set of integers $\mathbb{Z}$ defined by : $aRb$ iff $ab$ is a perfect square. Is $R$ an equivalence relation? Justify.

(b) If the function $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$, then find $f^{-1}(17)$.

(c) Use congruence to find the unit digit in $3^{100}$.

(d) If $p$ be a prime number, then find the value of $\phi(p)$.

**Answer *any four* questions :**

**4.** (a) Solve the system of linear congruences by Chinese remainder theorem.

$$x \equiv 2 \pmod 5, \quad x \equiv 3 \pmod 7, \quad x \equiv 5 \pmod 8$$

**5**

(b) Using principle of Mathematical Induction, prove that $(10^{2n-1}+1)$ is divisible by 11 $\forall n \in \mathbb{N}$. **5**

(c) Use Euclidean algorithm to find integers $u$ and $v$ such that $\gcd(20, 63) = 20u + 63v$. **5**

(d) If $p$ and $p^2+8$ are both prime numbers, then prove that $p = 3$. **5**

(e) Are the following relations partial order? Justify :

(i) Relation $R$ on $\mathbb{Z}$ defined by $aRb$ iff $a < b$

(ii) Relation $\rho$ on $\mathbb{Z}$ defined by $a\rho b$ iff $a \leq b$. **2+3**

(f) Show that $f : \mathbb{R} \to (-1, 1)$, $f(x) = \dfrac{x}{1+x^2}$, $x \in \mathbb{R}$ is neither injective nor surjective mapping. **5**

(g) Let $A = \mathbb{R} - \left\{-\frac{1}{2}\right\}$, $B = \mathbb{R} - \left\{\frac{1}{2}\right\}$ and $f : A \to B$ is defined by $f(x) = \dfrac{x-3}{2x+1}$, $\forall x \in A$. Does $f^{-1}$ exist? Justify your answer. **5**

## Group - C

**5. Answer *any two* questions :** $2\frac{1}{2} \times 2$

(a) Find a non-null matrix $A$ such that $A^2 = 0$, where 0 is the $2 \times 2$ zero matrix.

(b) For what values of $k$ the system of equations $kx + y + z = 1$, $x + ky + z = 1$, $x + y + kz = 1$ will have a unique solution?

(c) Does the rank of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ \alpha & 1 & \beta \\ 0 & 0 & 0 \end{bmatrix}$ depend on $\alpha$ and $\beta$? Justify.

(d)

$$V = \left\{ \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} : x_1, x_2, x_3, x_4 \in \mathbb{R} \right\}$$

is a Vector space over $\mathbb{R}$. Prove that

$$U = \left\{ \begin{bmatrix} x_1 & x_2 \\ x_2 & x_4 \end{bmatrix} : x_1, x_2, x_4 \in \mathbb{R} \right\}$$

is a subspace of $V$.

**Answer *any four* questions :** $5 \times 4$

**6.** (a) Reduce $\begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$ to Echelon matrix equivalent form.

(b) Prove that a set of more than $n$ vectors is linearly dependent in an $n$ dimensional vector space.

(c) Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. Find $k$ for which $AB = BA$.

(d) Determine the condition for which the system $x+y+z=1, x+2y-z=b, 5x+7y+az=b^2$ has (i) unique solution, (ii) no solution, (iii) many solutions.

(e) Extend the set $\{(2,1,1),(1,1,1)\}$ to a basis of $\mathbb{R}^3$.

(f) If $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, find the rank of the matrix $A + A^2 + A^3$.

(g) Find the independent vectors of the subset $W$ of $\mathbb{R}^3$, where $W = \{(x,y,z) \in \mathbb{R}^3 : x+y+z = 0\}$.

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