UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2025

MATHEMATICS — MINOR

Paper: MN-4

(Mechanics)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer *any six* questions : 2×6

(a) State the principle of conservation of linear momentum.

(b) The velocity of a particle parallel to x-axis and y-axis are $(a + cy)$ and $(b + dx)$ respectively, where a, b, c, d are constants. Find the path of the particle.

(c) What do you mean by apse and apsidal distance in a central orbit?

(d) State the Kepler's Laws of planetary motion.

(e) Forces equal to $\sqrt{2}P$, $\sqrt{3}P$ and Q acting on a particle are in equilibrium; find the angle between the forces $\sqrt{3}P$ and Q if the angle between $\sqrt{2}P$ and Q is 60°.

(f) Find the work done by gravity on a stone having a mass of $\frac{1}{2}$ kg during the tenth second of its fall from rest.

(g) A force of 20N acts on a body for 0.5 sec. Find the impulse of the force.

(h) A particle describes the curve $r = ae^{\theta}$ in such a manner that its acceleration has no radial component. Prove that its angular velocity is constant.

(i) State the Varignon's theorem.

(j) If time t be regarded as a function of velocity v prove that the rate of decrease of acceleration f is $f^3 \dfrac{d^2t}{dv^2}$.

Group - B

2. Answer *any seven* questions : 6×7

(a) Three forces P, Q, R acting at the vertices A, B, C respectively of a triangle, each perpendicular to opposite side, keep it in equilibrium. Prove that $P : Q : R = a : b : c$.

(b) A particle moves with an acceleration $(\mu \div \text{distance})$ towards a fixed point O. If it starts from rest at a distance 'a' from O, show that it will arrive at O in time $a\sqrt{\dfrac{\pi}{2\mu}}$.

(c) If h be the height attained by a particle when projected with a velocity V from the Earth's surface supposing its attraction constant and H be the corresponding height when the variation of gravity is taken into account, prove that $\dfrac{1}{h} - \dfrac{1}{H} = \dfrac{1}{r}$, where r is the radius of the Earth.

(d) A particle moving in a straight line subject to a resistance which produces the retardation kv^3, where v is the velocity and k is constant. If u is the initial velocity, then show that $v = \dfrac{u}{1+ksu}$, where s is the distance.

(e) Find the horse power of an engine which can project 10000 lbs of water per minute with a velocity of 80 ft/sec.

(f) A particle falls under gravity from a given height. Prove that at any moment of its motion the sum of kinetic energy and potential energy is constant.

(g) A bullet of mass 'm' moving with velocity v strikes a block of mass M which is free to move in the direction of motion of the bullet and is embedded in it. Show that a portion $\dfrac{M}{M+m}$ of the K.E. is lost.

(h) Two weights W and W' are connected by a light string passing over a smooth pulley. If the pulley moves vertically upwards with an acceleration equal to that of gravity, show that the tension of the string is $\dfrac{4WW'}{W+W'}$.

(i) A particle is projected vertically upwards under gravity with a velocity V. Assuming that the resistance of air is kv^2 per unit mass, where v is the velocity of the particle and k a constant, obtain the equation of motion of the particle and find the height of the particle where it will be at rest. If instead of being projected upwards, the particle falls downwards from rest, find the distance traversed by the particle in time t.

(j) Three forces P, Q, R act along the sides of BC, CA, AB of a triangle ABC taken in order. Show that if the resultant passes through the incentre, then $P+Q+R=0$ and the circumcentre, then $P\cos A + Q\cos B + R\cos C = 0$.

(k) A shell of mass m is ejected from a gun of mass M by an explosion which generated kinetic energy E. Prove that the initial velocity of the shell is $\sqrt{\dfrac{2ME}{(M+m)m}}$.

Group - C

3. Answer *any three* questions : 7×3

(a) Prove that a body projected from the Earth's surface with speed exceeding 7 miles per second will not return to the Earth.

(b) If a planet is suddenly stopped in its orbit, supposed circular, show that it would fall into the Sun in a time which is $\dfrac{\sqrt{2}}{8}$ times the period of the planet's revolution.

(c) A particle describes a parabola under a force which is always directed perpendicularly towards its axis. Prove that the force must be inversely proportional to the cube of the ordinate.

(d) Find the tangential and normal components of velocity and acceleration of a particle which describes a plane curve.

(e) If a particle describes a parabola $x^2 = 8y$ under a force always perpendicular to the y axis. Find the law of force and the velocity at any point of its orbit.

(f) If v_1 and v_2 are the linear velocities of a planet when it is respectively at nearest and farthest from the Sun, then prove that $(1-e)v_1 = (1+e)v_2$, where e is the eccentricity of the elliptic orbit of the planet.