UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2026

MATHEMATICS — HONOURS

Paper: DSCC-2

(Basic Algebra)

Full Marks: 75

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise: https://palmaths.com/cu-pyqs/2026/maths-major-sem-2-dscc-2/

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and Symbols have their usual meaning.

Group - A

1. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) Show that the sum of 99th powers of the roots of the equation $x^7 - 1 = 0$ is zero.

(b) For positive real numbers a, b, c, find the least value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

(c) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then prove that $x_1 x_2 x_3 \cdots$ to $\infty = 1$. *[PALMATHS note: As printed in the CU paper; the product actually equals -1.]*

2. Answer *any four* questions : 5×4

(a) Show that the solutions of the equation $(1+x)^{2n} + (1-x)^{2n} = 0$ are

$$x = \pm i \tan\frac{(2k-1)\pi}{4n}, \quad k = 1, 2, \ldots, n.$$

(b) If $\tan^{-1}(x+iy) = u - iv$, where x, y, u, v are real numbers and $(x, y) \neq (0, \pm 1)$, then prove that $x^2 + y^2 + 2x\cot 2u = 1$ and $x^2 + y^2 + 1 + 2y\coth 2v = 0$.

(c) If $a_1, a_2, \ldots, a_n$ are n positive rational numbers and $S = a_1 + a_2 + \cdots + a_n$, then prove that

$$\left(\frac{S}{a_1} - 1\right)^{a_1}\left(\frac{S}{a_2} - 1\right)^{a_2} \cdots \left(\frac{S}{a_n} - 1\right)^{a_n} \leq (n-1)^S.$$

(d) Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero.

(e) State Descartes' rule of signs regarding the number of positive roots of an equation with real coefficients. Apply this rule to show that the equation $x^4 + 12x - 5 = 0$ has two real roots and two non-real roots.

(f) Solve the equation $x^3 - 18x - 35 = 0$, by Cardan's method.

Group - B

3. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) If $S = \{1, 2, 3, 4\}$, then write an equivalence relation on S and also write the partition corresponding to the equivalence relation.

(b) Is the mapping $f : \mathbb{R} \to (-1,1)$ defined by $f(x) = \dfrac{1}{1+x^2}$, $x \in \mathbb{R}$ a bijective mapping? Justify your answer.
$\mathbb{R}$ = the set of real numbers and $(-1,1) = \{x \in \mathbb{R} \mid -1 < x < 1\}$.

(c) Prove that the product of any three consecutive integers is divisible by 6.

4. Answer *any four* questions :

(a) (i) Let $f, g : A \to B$ and $h : B \to C$ be mappings. If $h \circ f = h \circ g$ and h is injective, then prove that $f = g$.

(ii) Let $f : X \to Y$ be a function and $B \subseteq Y$. Prove that $f\left(f^{-1}(B)\right) \subseteq B$. **3+2**

(b) Prove that the equation $ax + by = c$ has integral solution if $\gcd(a,b)$ divides c where a, b, c are integers such that a and b are not both zero. **5**

(c) Solve by Chinese remainder theorem : $x \equiv 2 \pmod 7$, $x \equiv 3 \pmod 9$, $x \equiv 2 \pmod{11}$. **5**

(d) (i) Let ρ be an equivalence relation on a set S and $a, b \in S$. Show that $cl(a) = cl(b)$ if and only if $a \rho b$.

(ii) Find $\varphi(2400)$. **3+2**

(e) (i) If p is a prime number such that $p \mid ab$, for some positive integer a, b, then prove that either $p \mid a$ or $p \mid b$.

(ii) Find the remainder when $1! + 2! + \cdots + 50!$ is divided by 15. **2+3**

(f) If a and b are relatively primes, then $\gcd[(a+b),(a-b)] = 2$ or 1. **5**

Group - C

5. Answer *any two* questions : $2\frac{1}{2} \times 2$

(a) If $x \neq 0$ and rank $\begin{bmatrix} x & y & z \\ y & z & t \end{bmatrix} = 1$, prove that $x : y : z : t = 1 : \lambda : \lambda^2 : \lambda^3$ for some λ.

(b) Examine linear dependence of the set of vectors $\{(1,-1,2,4),(2,-1,5,7),(-1,3,1,-2)\}$ in $\mathbb{R}^4$.

(c) Show that the following system of equations has infinite number of solutions :

$$x + 4y + 2z = 1, \quad 2x + 7y + 5z = 1, \quad 4x + 14y + 10z = 2.$$

6. Answer *any four* questions :

(a) (i) Is the system of equations $x + y - z = 6$, $2x - 3y + z = 1$, $2x - 4y + 2z = 1$ solvable? If so, solve it.

(ii) If $(0,2,a)$ can be expressed as $(0,2,a) = b(1,2,2) + c(1,0,-1)$, find the values of a, b, c. **3+2**

(b) (i) If the sum of each row of an $n \times n$ non-singular matrix A is $k\,(\neq 0)$, then prove that the sum of each row of A^{-1} is $\dfrac{1}{k}$.

(ii) Find the rank of the matrix $adj(A)$, where A is a 4×4 matrix of rank two. **4+1**

(c) Find the values of λ and μ for which the system of equations :

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

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have (i) no solution (ii) unique solution and (iii) infinitely many solutions. **5**

(d) Find the row reduced echelon form of $A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{pmatrix}$ and hence find the rank of it. **5**

(e) Solve the system of equations :

$$\begin{aligned} x + 2y + z - 3w &= 1 \\ 2x + 4y + 3z + w &= 3 \\ 3x + 6y + 4z - 2w &= 4. \end{aligned}$$

5

(f) Show that the set $\{(1,1,0),(1,0,1),(0,1,1),(1,1,1)\}$ spans $\mathbb{R}^3$ but not linearly independent in $\mathbb{R}^3$. **5**

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