UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2026

MATHEMATICS — MDC

Paper: MN-5

(Advanced Calculus)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Throughout the question $\mathbb{R}$ denotes the set of real numbers and $\mathbb{N}$ denotes the set of natural numbers.

Group - A
(Marks : 20)

1. Answer *any four* questions :

(a) Define monotone sequence of real numbers. Examine whether the sequence $a_n = \dfrac{3n+1}{2n+5}$, $n \in \mathbb{N}$ is convergent. **2+3**

(b) Prove that a Cauchy sequence of real numbers is bounded. Is converse true? Justify your answer. **2+1+2**

(c) State Cauchy's general principle of convergence of a sequence of real numbers. Prove that the sequence $\{a_n\}_{n=1}^{\infty}$, is not convergent, where $a_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \cdots + \dfrac{1}{n}$, $n \in \mathbb{N}$. **2+3**

(d) Examine the convergence of the series $\dfrac{1}{1\cdot 3} + \dfrac{2}{3\cdot 5} + \dfrac{3}{5\cdot 7} + \dfrac{4}{7\cdot 9} + \ldots$ **5**

(e) Examine the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. **5**

(f) Test the convergence of the series $1 + \dfrac{1}{1!} + \dfrac{2^2}{2!} + \dfrac{3^3}{3!} + \ldots$ **5**

(g) State Leibnitz's theorem for convergence of an alternating series. Investigate the convergence of $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$. **2+3**

Group - B
(Marks : 25)

2. Answer *any five* questions :

(a) State the Cauchy's principle for the existence of $\lim\limits_{x\to c} f(x)$, where $f : D \subset \mathbb{R} \to \mathbb{R}$ be a function, $c \in D'$. Hence show that $\lim\limits_{x\to 0} \cos\dfrac{1}{x}$ does not exist. **2+3**

(b) Evaluate : (i) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin^2 x}$ (ii) $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$. **3+2**

(c) Find the value of 'a' for which the function $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ is continuous at the point $x = 3$. **5**

(d) Let $f : [a,b] \to \mathbb{R}$ and $g : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and let $f(a) < g(a)$, $f(b) > g(b)$. Show that there exists a point c in (a,b) such that $f(c) = g(c)$. **5**

(e) Let $f(x) = \begin{cases} x\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Check whether the function is continuous at $x = 0$. Also investigate for differentiability of the function at $x = 0$. **2+3**

(f) Show that $\dfrac{x}{1+x} < \log(1+x) < x$ for, $\forall x > 0$. **5**

(g) State Rolle's Theorem. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 10$ on $[2,3]$. **5**

(h) State with reason whether the inverse of the function $f(x) = x^3 + 3x$, $\forall x \in \mathbb{R}$ exists. Also check whether the inverse function is continuous in $\mathbb{R}$. Justify your answer. **3+2**

(i) Show that Maclaurin's infinite series expansion of the function e^x is given by :

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \ldots \quad \text{for all } x \in \mathbb{R}.$$

5

Group - C
(Marks : 30)

3. Answer *any six* questions :

(a) Let $f(x,y) = \dfrac{x^2y}{x^4+y^2}$, $(x,y) \neq (0,0)$. Discuss the existence of the limit of $f(x,y)$ as $(x,y) \to (0,0)$. **5**

(b) Prove that the function

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at the point $(0,0)$. **5**

(c) Calculate partial derivatives f_x and f_y at $(0,0)$ of the function $f(x,y) = \sqrt{|xy|}$. Is the function differentiable at $(0,0)$? Justify your answer. **3+2**

(d) Let $V = \log(x^3+y^3+z^3-3xyz)$, then prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}\right) V = -\dfrac{3}{(x+y+z)^2}$. **5**

(e) State Schwarz's theorem for equality of mixed second order partial derivatives. Check whether $f_{xy} = f_{yx}$ at $(0,0)$ for the function $f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. **2+3**

(f) If $V = \cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then verify that $\cos V$ is a homogeneous function of x and y of degree $\frac{1}{2}$. Hence prove $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} + \dfrac{1}{2}\cot V = 0$. **5**

(g) Given that z is a function of x, y and $x = u+v$, $y = uv$, prove that

$$\frac{\partial^2 z}{\partial u^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} = (x^2 - 4y)\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial z}{\partial y}.$$

5

(h) Find $\frac{dz}{dt}$ at $t = 2$, when $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$. **5**

(i) Let z be a differentiable function of x and y. If $x = r\cos\theta$ and $y = r\sin\theta$, then prove that

$$\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}.$$

5

(j) If assume that the function $f(x, y) = y^2 - yx^2 - 2x^5 = 0$ uniquely defines y as a function of x, then find $\frac{dy}{dx}$ at $(1, -1)$. **5**

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