UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2026

MATHEMATICS — MINOR

Paper: MN-2

(Basic Algebra)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Throughout the question paper $\mathbb{R}$ denotes the set of real numbers. Other symbols have their usual meaning.

Group - A
(Marks - 25)

1. Answer *any five* questions :

(a) (i) Find the values of $(i-1)^{\frac{1}{5}}$.

(ii) If $a+\frac{1}{a}=2\cos\alpha$, then prove that $a^n+\frac{1}{a^n}=2\cos n\alpha$, where n is an integer. **2+3**

(b) Prove that, $\sin\left[i\log\frac{a-ib}{a+ib}\right]=\frac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero. **5**

(c) (i) Find the principal values of $\text{Log}\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$.

(ii) Apply Descartes' rule of signs to find the number of real roots of the equation $x^6-1=0$. **3+2**

(d) Solve, by Cardan's method : $x^3-12x+65=0$. **5**

(e) If one root of the equation $x^3+px^2+qx+r=0$ is equal to the sum of the other two roots, then prove that $p^3+8r=4pq$. **5**

(f) Using Sturm's theorem, find the nature and positions of the roots of the equation $x^3-7x+2=0$. **5**

(g) Prove that $(a+b+c)^2\geq 3(ab+bc+ca)$, where a, b, c are three real numbers. **5**

(h) Show that $n^n>1\cdot 3\cdot 5\cdots(2n-1)$, where n is a positive integer greater than 1. **5**

(i) Solve the equation $x^3-9x^2+23x-15=0$, if two of the roots are in the ratio $3:5$. **5**

Group - B
(Marks - 25)

2. Answer *any five* questions :

(a) If two mappings $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^2$ and $g(x)=x-2$, then verify the equality of $f\circ g$ and $g\circ f$. **5**

(b) Prove or disprove : $\rho=\{(a,b)\in\mathbb{Z}\times\mathbb{Z}: 3a+4b \text{ is divisible by } 7\}$ is an equivalence relation on $\mathbb{Z}$. **5**

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(c) Show that the function $f : \mathbb{R} - \{\sqrt{2}, -\sqrt{2}\} \to \mathbb{R}$ defined by,

$$f(x) = \frac{x}{x^2 - 2}$$

is surjective, but not bijective. **3+2**

(d) Use Euclidean algorithm to find integers u and v such that $\gcd(252, 595) = 252u + 595v$. **5**

(e) Solve for x using Chinese remainder theorem :

$$x \equiv 3 \pmod 5, \quad x \equiv 4 \pmod 7, \quad x \equiv 6 \pmod 9.$$

5

(f) Prove that $3^{2n} - 8n - 1$ is divisible by 64 for all $n \in \mathbb{N}$. **5**

(g) If both of p and $p^2 + 8$ are prime numbers, prove that $p = 3$. **5**

(h) Prove that $\varphi(3n) = 3\varphi(n)$ if and only if 3 is divisor of n, where φ is the Euler's Phi function. **5**

(i) (i) Prove that $11^{34} \equiv 2 \pmod{17}$.
(ii) If $\gcd(a, b) = 1$, prove that $\gcd(a + b, a) = 1$. **3+2**

Group - C
(Marks - 25)

3. Answer *any five* questions :

(a) Find the rank of the matrix

$$\begin{pmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{pmatrix}$$

by using elementary row operation. **5**

(b) Investigate for what values of β and μ the following equations

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \beta z = \mu$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **2+2+1**

(c) If $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, find Rank $(A + B)$. **5**

(d) If (α, β, γ) is a linearly independent set of vectors in $\mathbb{R}^3$, then prove that $\{\alpha + \beta + \gamma,\ \beta + \gamma,\ \gamma\}$ is also linearly independent. **5**

(e) Reduce the following matrix to a row reduced echelon matrix :

$$A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ 2 & 5 & 3 & 9 \\ 3 & 8 & 5 & 14 \end{pmatrix}$$

5

(f) Find a Spanning set of the subset : $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$. **5**

(g) (i) Determine the values of k so that the rank of the matrix $\begin{pmatrix} 1 & k \\ 2 & k^2 \end{pmatrix}$ is 1.

(ii) Prove or disprove : the set $\{(2,3,1),(2,1,3),(1,1,1)\}$ is linearly dependent in $\mathbb{R}^3$. **3+2**

(h) Express the equations :

$$\begin{aligned} x+2y+z &= 1 \\ 2x+4y+3z &= 3 \\ 3x+6y+4z &= 4 \end{aligned}$$

in the form $AX = B$ and solve. **2+3**

(i) (i) Find the value of λ so that the matrix $A = \begin{pmatrix} 1 & 0 \\ 1 & \lambda \end{pmatrix}$ is equal to its own inverse.

(ii) Express $(-1,2,4)$ as linear combination of $(-1,2,0)$, $(0,-1,1)$ and $(3,-4,2)$ over $\mathbb{R}^3$. **2+3**

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