UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2026

MATHEMATICS — MINOR

Paper: MN-2

(Basic Algebra)

Full Marks: 75

Typed & checked by PALMATHS (Biswadip Sir, Garia, Kolkata) from the original CU paper · More papers year-wise:
https://palmaths.com/cu-pyqs/2026/maths-minor-sem-4-mn-2/

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols have their usual notation.

Group - A

1. Answer *any five* questions : 5×5

(a) Show that the product of all the values of $(1+i\sqrt{3})^{\frac{3}{4}}$ is 8.

(b) Let α, β, γ be three roots of the equation $x^3+2x^2+3x+1=0$. Find the equation whose roots are $\alpha(\beta+\gamma)$, $\beta(\gamma+\alpha)$, $\gamma(\alpha+\beta)$.

(c) Using Descartes Rule of Sign, state possible number of positive, negative and imaginary roots of the equation $x^5+2x^4+7x^3+9x^2-3x-9=0$.

(d) If a, b, c, d are four positive numbers, prove that $(a+b+c+d)(a^3+b^3+c^3+d^3) \geq 16abcd$.

(e) If $a+\dfrac{1}{a}=2\cos\alpha$, then prove that $a^n+\dfrac{1}{a^n}=2\cos n\alpha$.

(f) The product of two roots of the equation $x^4+x^3+2x^2+2x+4=0$ is equal to the product of the other two roots. Find the roots.

(g) Solve the equation $x^3-12x+65=0$ by Cardan's method.

(h) Solve the equation by Ferrari's method $x^4-10x^3+35x^2-50x+24=0$.

(i) Prove that $\sin\left[i\log\dfrac{a-ib}{a+ib}\right]=\dfrac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero.

Group - B

2. Answer *any five* questions :

(a) Examine if the relation R on the set $\mathbb{Z}$ is an equivalence relation;

$$R=\{(a,b)\in\mathbb{Z}\times\mathbb{Z} : 2a+3b \text{ is divisible by } 5\}.$$

5

(b) Solve the following system of congruences :

$$x\equiv 2 \pmod 7, \quad x\equiv 5 \pmod{19}, \quad x\equiv 4 \pmod{15}.$$

5

(c) Use mathematical induction to prove the sum of first n natural number is $\dfrac{n(n+1)}{2}$. **5**

(d) Let $f : A \to B$ and $g : B \to C$ be two mapping. If $g \circ f : A \to C$ is bijective, prove that f is injective and g is surjective. **5**

(e) (i) Find all solution of $5x \equiv 3 \pmod{19}$.

(ii) If n is a positive integer such that $n^3 + 1$ is a prime number, show that $n = 1$. **3+2**

(f) Let $A = \mathbb{R} \setminus \{3\}$ and $B = \mathbb{R} \setminus \{1\}$. Define $f : A \to B$ by $f(x) = \dfrac{x-2}{x-3}$. Show that f is one to one and onto. **5**

(g) If p and $p^2 + 8$ are both prime numbers, prove that $p = 3$. **5**

(h) A relation ρ is defined on $\mathbb{Z}$ by $x\rho y$ holds if and only if $4x + 7y$ is divisible by 11 for $x, y \in \mathbb{Z}$. Show whether ρ is an equivalence relation on $\mathbb{Z}$ or not. **5**

(i) Show that the functions $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{1\}$ and $g : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{2\}$ defined by $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{2x}{x-1}$ are inverses to each other. **5**

Group - C

3. Answer *any five* questions :

(a) Solve the following system of linear equation :

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

5

(b) Find the rank of the matrix by reducing it to row reduced echelon form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}.$$

5

(c) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, then show that $A^2 - 4A - 5I = 0$. **5**

(d) Find a spanning set of $\mathbb{R}^3$ containing the vectors $(1, 2, 1)$ and $(2, 1, 1)$. **5**

(e) Find the dimension and basis of the solution space of the system of equations :

$$x - 2y + z = 0, \qquad x - 2y - z = 0.$$

5

(f) For what values of h and k, the system of equations

$$x_1 + 2x_2 + x_3 = 8, \quad 2x_1 + x_2 + 3x_3 = 13, \quad 3x_1 + 4x_2 - hx_3 = k$$

has (i) unique solution, (ii) no solution, (iii) an infinite number of solutions. **5**

(g) For what real values of k, the set $S = \{(k, 1, k), (0, k, 1), (1, 1, 1)\}$ form a basis of $\mathbb{R}^3$. **5**

(h) (i) Find the value of λ so that the matrix $A = \begin{pmatrix} 1 & 0 \\ 1 & \lambda \end{pmatrix}$ is its own inverse.

(ii) If $\alpha = (1,1,2)$, $\beta = (0,2,1)$, $\gamma = (2,2,4)$, determine whether β is a linear combination of γ and α. **3+2**

(i) If $P = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$; then find rank of $P+Q$. **5**

PALMATHS