UNIVERSITY OF CALCUTTA · Previous Year Question Paper

2026

MATHEMATICS — MINOR

Paper: MN-3

(Ordinary Differential Equation - I and Group Theory - I)

Full Marks: 75

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The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols and Notations have their usual meanings.

Group - A

[Ordinary Differential Equation – I]
(Marks : 45)

Answer *any nine* questions.

1. (a) Find the order and degree of the differential equation

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = a\frac{d^2y}{dx^2}.$$

(b) Obtain the differential equation of all circles each of which touches the axis of x at the origin. **2+3**

2. Solve : $(3x+2y-5)\,dx+(2x+3y-5)\,dy=0$. **5**

3. Solve $y-x\frac{dy}{dx}=2\left(1+x^2\frac{dy}{dx}\right)$; given $y=1$ when $x=1$. **5**

4. Obtain the general and singular solution of $yp=xp^2-p-2$, $\left(p\equiv\frac{dy}{dx}\right)$. **5**

5. Solve : $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$. **5**

6. Solve $\frac{d^2y}{dx^2}+y=\tan x$ using variation of parameters. **5**

7. Use the method of Undetermined Coefficients to find a General solution : $y''+9y=x\sin(2x)$. **5**

8. Solve : $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^3+\frac{dy}{dx}$. **5**

9. Solve : $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=e^{3x}$. **5**

10. Show that $y=x^m$ is a non-trivial solution of $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$ if $m(m-1)+Pmx+Qx^2=0$.
Hence find the condition for which $y=\frac{1}{x}$ is a solution of the same ordinary differential equation. **3+2**

11. Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0, \qquad \frac{dy}{dt} + 5x + 3y = 0.$$

5

12. Solve : $\sin x \frac{d^2y}{dx^2} - \cos x \frac{dy}{dx} + 2y \sin x = 0.$ **5**

13. Obtain the singular solution : $y = px + \frac{a}{p}$ $\left(\text{where } p = \frac{dy}{dx}\right)$. **5**

14. Solve : $y\left\{x(2x+1)\frac{dy}{dx} - y\left(\frac{dy}{dx}\right)^2\right\} = 2x^3.$ **5**

15. Find the integrating factor of the equation $xy\,dx + (2x^2 + 3y^2 - 20)\,dy = 0$, and hence solve it. **1+4**

16. Solve the equation $y(2xy+1)\,dx + x\left(1 + 2xy + x^2y^2\right)dy = 0$ by substitution $xy = v$. **5**

Group - B

[Group Theory – I]
(Marks : 30)

Answer *any six* questions.

17. For any two elements a and b of a group G, show that $(ab)^2 = a^2b^2$ if and only if $ab = ba$ in G. **5**

18. Prove that a group of prime order is Cyclic. Give an example of a finite group which is not Cyclic, but whose all proper subgroups are Cyclic. Justify your answer. **3+2**

19. Let $(G, \circ)$ be a finite cyclic group generated by a. Then prove that $o(G) = n$ iff $o(a) = n$. **5**

20. If $\alpha = (1\,2\,3)(4\,5)$ and $\beta = (1\,4)(2\,5)$, then find $\beta^{-1}\alpha\beta$. **5**

21. If in a group G, $a^5 = e$ and $aba^{-1} = b^2\ \forall a, b \in G$, find the order of b. **5**

22. Let $G = \mathbb{R} - \{0\}$ and $a * b = \frac{ab}{2}$, show that $(G, *)$ is an abelian group. **5**

23. Show that every finite group of order less than six must be abelian. **5**

24. Let $(G, \circ)$ be a group and $a, x \in G$. Then show that $O(a)$ and $O(x^{-1} \circ a \circ x)$ are same. **5**

25. Show that the set $S = \{1, i, -1, -i\}$ is a commutative group with respect to the multiplication of complex numbers. **5**

26. Prove that the set of rational numbers, other than 1, forms a commutative group with respect to the composition $*$ defined by $a * b = a + b - ab$ for $a, b \in \mathbb{Q} - \{1\}$. **5**

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